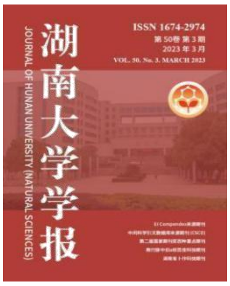

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The Modified EOQ-Formula for Deferred Order Payments and Delays in Receipt of Revenue

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Abstract: The purpose of this article is to analyse the optimisation strategy of inventory management, considering the possibility of deferred payments, delays in receiving revenue and vehicle capacity restrictions. The study is based on the modification development for the multiproduct EOQ-model that acknowledges the following factors: 1) the time value of money; 2) the possibility of deferred payment negotiated prior to placing an order; 3) specific nature of incoming payments with receipt of revenues from goods subsequent to their delivery; 4) the vehicle capacity; 5) the case of a company paying for its orders with revenues from the goods delivered. This article establishes sufficient conditions to be imposed on the length of deferred payment for the order and the acceptable delay in payment for goods making it possible to form required payments from the proceeds.

Keywords: supply chain optimization, inventory management, time value of money, deferred order payment, vehicle load capacity.

订单延期付款和收款的EOQ (经济订货量) 修改后模型

摘要：本文的目的是分析库存管理的优化策略，考虑延期付款、延期收款和车辆装载量限制的可能性。 该研究基于对多产品的 EOQ 修改开发后模型，该模型考虑以下因素：1) 货币的时间价值； 2) 下订单前协商延期付款的可能性； 3) 在货物交付后收到货款的具体性质； 4) 车辆装载量； 5) 公司用交付货物的货款支付订单的情况。 本条规定了对订单延期付款的期限和可接受的货物延期付款所需的充分条件，从而保证收益可担负所需的货款。

关键词：供应链优化，库存管理，货币时间价值，延期订单支付，车辆装载量.

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Introduction

The time value of money (TVM) and the cargo/load capacity of vehicles utilised are not considered in classical inventory management theory, making it impossible to factor in the corresponding specificities of cash flows modelled during the supply chain optimisation process.

Some articles have already provided a particular approach in optimising inventory in EOQ-models using TVM based on simple interest. For traditional models, this approach has already been presented [1]. Article [2] investigated the specifics of inventory management with regard to deferred payment of orders. A special modification of the EOQ-model [3] established the optimum supply strategy for orders that are paid for in advance.

The purpose of this article is to provide new iterations of the EOQ-model that considers the following set of factors that are important for practical applications:

1) the receipt of payment for each product, and consequently all revenue, which typically implies a certain delay by the pre-determined period of time;
2) the permissible delay in payments;
3) weight and dimensions characteristics of a product and the load capacity of vehicles.

Deferred payment of the order and delayed receipt of revenue are bound to affect the working capital profitability of these supply chains. TVM accounting procedures in this article are different from the approach proposed in most publications, where the continuously compound interest is employed for TVM calculations. This is prompted by the intention to incorporate interest procedures into the format of the relevant banking structures. This approach to optimisation does not allow for the cash flow performance of the supply chain itself to be considered, as it is not consistent with the principles of financial analysis and financial mathematics.

1. Theoretical Background

The specificity of this approach to supply optimisation is illustrated by the work [10] on modifying the EOQ-formula in light of inflation and the work [5] on modifying the Economic Production Quantity model considering the current value of all costs. Work [7] examining implications of the inflation and the time value of money for inventory models with linear time-dependent demand, as well as in the paper [12] presenting EOQ-model with the constant demand, the time value of money and additional factors like the damage of goods in storage and delayed payments.

Supply optimisation for inventory management with TVM being introduced as the continuously compounded interest is presented in numerous papers. For example, study [6] utilised discounted cash flow for the EOQ-model where the supplier provides various trading credits. A generalisation based on discounted cash flow and two levels of trade loans is presented in the paper [13]. The article [14] presented the model considering TVM with deferred payments for damaged products. The study [8] illustrated an inventory management model based on the TVM with a normal distribution of the demand profile. [11] proposed an EOQ-model considering stock-dependent demand and two levels of trade credit along with TVM.

A different approach to TVM is required to analyse the efficiency of the cash flows of supply chain itself: TVM is implemented using an interest rate which characterises cash flows of the supply chain. Its value is not an exogenous factor, it is established based on the features of the modelled supply chain. For the new modification of the EOQ-formula the target function of the inventory optimisation problem involves minimising total costs of supply chain operation.

In addition, an issue in inventory management is the inclusion of vehicle cargo load capacity and shipping discounts in the EOQ calculations. [4] developed an economic lot size model that incorporates price-dependent demand along with quantity and freight discounts. Research [9] introduced a complex algorithm that simultaneously applies all-units quantity discounts on purchasing price and freight cost. Article [16] proposed algorithms for determining EOQ with either all-weight or incremental freight discounts. [15] developed decomposition rules for breaking down incremental quantity and weight discounts into other discount scenarios.

2. Attributes of Inventory Management Model Involving TVM

Inventory optimisation procedure involving TVM requires allocation of all cash flows to specific moments of time during the "reorder interval" (period of time between deliveries). Thereafter, one considers the EOQ-model's form, which allows order payment deferral time of Δ_P (years). In addition, the model considers that payment for goods sold is delayed by Ω (years). The peculiarity of the timing of revenue receipts for the EOQ-model including TMV excluding delayed payments for the goods sold is emphasised as follows: when making provisions for the TVM based on simple interest, the moment of receipt of the entire amount of proceeds from the fulfilment of the order of goods, on average, must correlate with the midpoint of the reorder interval.

The moment of receipt of total proceeds from the implementation of the order, on average, will not correlate with the middle of the re-order interval, but with the moment that is delayed relative to it by the time Ω. Then, it is assumed that payments for storage costs and other necessary business expenditures are also performed with a corresponding delay relative to the midpoint of the re-order interval.

Indication of the notations used:

- i – the range of products ($i = 1, 2, \dots, n$);
- D_i – the annual consumption of i-product (units of goods);
- C_0 – the cost of the delivery of one cargo lot;
- C_{Pi} – the cost of the i-product per unit;
- P_{Pi} – the profit from the sale of the i-products per unit;
- L_{Pi} – charges per unit of the i-product required to maintain the business;
- C_{hi} – annual costs of storage per unit of the i-product;
- q_i – the size of the i-order, i.e., the number of the i-product units in a purchase order, which is an optimised value (units of goods);
- T – a period between deliveries, linked to the q_i by the equation $T = q_i/D_i$, an optimised value (years);
- r_M – an annual rate that reflects the conversion of the working capital into profit for the supply chain (if optimisation process involves TVM);

The following factors are considered for the simulated supply chain cash flows:

- The payment for an order is implemented after delivery with a delay time of Δ_P;
- The receipt of revenue from the sale of goods on average, is correlated with a delay of Ω relative to the middle of the re-order interval;
- The payment of storage costs is correlated, on average, with the specified time $T/2 + \Omega$;
- Additional business expenses L_{Pi} are correlated with the specified time $T/2 + \Omega$.

The following features of the model should also be noted:

- Payments for the cost of delivery C_0 correspond with the beginning of each re-order interval;
- Order payment, considering the postponement Δ_P (relative to the beginning of each reorder interval), is $\Sigma q_i \cdot C_{Pi}$. If this amount has to correlate to the beginning of the period in the optimisation procedures, it has to be considered with a discount that depends on the duration of Δ_P. The model assumes that revenues cover the specified order payment during the corresponding re-order interval;
- Payments for business-related deductions (L_{Pi}) together with storage costs, that are correlated with the moment of profit receipt correspond with the following equation: $\Sigma q_i \cdot L_{Pi} + \Sigma C_{hi} \cdot q_i \cdot T/2$. These payments correlate with the point of time $T/2 + \Omega$ for a reorder interval with T duration;
- The revenue for the entire order that is correlated, on average, with a delay of Ω, equals $\Sigma(C_{Pi} + P_{Pi}) \cdot q_i$.

The key criterion of the optimisation problem is minimising the total annual supply chain operating costs. While determining these costs, at the end of the year the outgoing cash flows are discounted by simple interest to a single point in time.

In the papers [1, 2] it has been demonstrated that a problem of this type is equivalent to the problem of selecting the time interval T between deliveries where the intensity of the revenue stream is maximised. The optimisation problem introduced here is presented in accordance with this approach.

3. Optimisation of Supply Strategy Parameters

The indicator of revenue flow intensity at the reorder interval can be determined by using the approach noted [1], namely, the problem of maximizing the following objective function $F_M \rightarrow max$, with a limitation of $T \geq 2\Omega$:

$$F_M = \frac{1}{T}[\sum q_i \cdot (C_{Pi} + P_{Pi} - L_{Pi} - C_{hi} \cdot T/2) - \{1 + r_M \cdot (T/2 + \Omega)\} \cdot (C_0 + d_P \cdot \sum q_i \cdot C_{Pi})], \quad (1)$$

where the restriction of $T \geq 2\Omega$ is required for simulating the moment of revenue receipt for the order, that on average, does not exceed the corresponding reorder interval.

The F_M function in the formula (1) shows that all cash flows in the reorder interval are accounted for in the bracketed expression. They are scaled to a single point in time within $T/2 + \Omega$ interval. The reduction procedures are implemented with respect to the delivery cost (C_0) and the ordering cost ($\Sigma q_i \cdot C_{Pi}$), recognising that the payment of this cost is deferred for a Δ_P period. Payments deferred by Δ_P are in turn further discounted to preliminarily reduce them to the origin of the specified interval. The discounting factor d_P is determined by the equation $d_P = 1/(1 + r_M \cdot \Delta_P)$.

Consider the optimal supply strategy for the specifics of the model. First, eliminating the exponents of q_i in the formula (1) based on the equality $T = q_i/D_i$. Then, introducing the scalar product $\vec{D} \cdot \vec{C}_h = \sum D_i C_{hi}$ of annual consumption, vector $\vec{D} = (D_1, D_2, \dots, D_n)$ and annual storage costs vector $\vec{C}_h = (C_{h1}, C_{h2}, \dots, C_{hn})$. Besides, one establishes $\vec{D} \cdot \vec{C}_P$ as the scalar product of the vector $\vec{D}$ and i-product cost vector $\vec{C}_P = (C_{P1}, C_{P2}, \dots, C_{Pn})$. Finally, changing the sign of the entire equation for F_M (this, of course, changes the direction of optimisation) and discards expressions that do not contain the variable T. Thus, the problem of inventory optimisation (1) can be represented as

$$\frac{2C_0 \cdot (1 + r_M \cdot \Omega)}{T} + T \cdot \vec{D} \cdot (\vec{C}_h + r_M \cdot d_P \cdot \vec{C}_P) \rightarrow min,$$

with a limitation of $T \geq 2\Omega$.

Among two terms for the target function, when T is a variable, one presents a hyperbola and the other is a line extending from the origin. First, a new modified EOQ-formula for the minimum point T_0 of the specified target function is readily derived, which corresponds to the condition where the constraint of $T \geq 2\Omega$ is not considered:

$$\mathrm{T}_0 = \sqrt{\frac{2C_0(1+r_M \cdot \Omega)}{\vec{D}(\vec{C}_h + r_M \cdot d_P \cdot \vec{C}_P)}}. \quad (2)$$

The optimum value T_{opt} considering the delays and deferrals for the modelled cash flows and TVM with limitation of $T \geq 2\Omega$ can be determined by the formula:

$$T_{opt} = \begin{cases} T_0 \; if \; 2\Omega \leq T_0; \\ 2\Omega - otherwise. \end{cases} \quad (3)$$

The size of the *i*-order denoted $q_{i\,opt}$ for these supplies is determined by the formula $q_{i\,opt} = D_i \cdot T_{opt}$.

The specification of the model, caused by order payment deferrals, is taken into account (2) by a factor in the form of $d_P = 1/(1 + r_M \cdot \Delta_P)$. This factor should be considered when calculating costs of goods' vector $\vec{C}_P$ that is in the denominator of the root equation in this formula. d_P is always less than 1.

Combined with (2), it substantiates the following peculiar results. The presence of permitted deferral of order payment (when TVM is involved in the optimisation procedure) results in a longer duration of time between deliveries and increased order size for each item (compared to the model without order payment deferrals, i.e. $\Delta_P = 0$).

The delays in payment for the goods, are taken into account (2) by the factor $(1 + r_M \cdot \Omega)$. It should be allocated to the index C_0 (delivery costs) in the numerator of the root expression. The value of this factor is always greater than 1. Evidently, the presence of delays in obtaining revenue also results (when optimising considering TVM) in a greater duration of the interval between deliveries.

4. Delays in Payments for Goods Limitations

As shown in (3), formula (2) determines the optimum reorder interval for the model in question, when the condition $T_0 \geq 2\Omega$ is fulfilled. This condition corresponds to the acceptable delay. This condition can be presented differently: in the form of a limitation for the allowable delay in payment for goods. By substituting T_0 in inequation $T_0 \geq 2\Omega$, inequation $2\Omega^2 \cdot \vec{D} \cdot (\vec{C}_h + r_M \cdot d_P \cdot \vec{C}_P) - C_0 r_M \Omega - C_0 \leq 0$ is obtained.

Considering this inequation relative to index $\Omega \geq 0$. It is essential to determine the extent of the delay in payment so that the moment of receipt of the proceeds, on average, does not exceed the reorder interval. On one hand, the layout of the left-hand side of the inequation graphically represents a parabola (with the variable Ω), with branches opening upwards. On the other hand, the discriminant of the quadratic equation (if the left-hand side is equal to zero) is always greater than zero. The aforementioned parabola intersects the x-axis (for the variable Ω) twice: one of these Ω values is in the area of negative values for the Ω variable. Therefore, one of the roots of the quadratic equation is negative and the other one is positive.

The structure of the interval of acceptable values for revenue delay Ω is as follows: $\Omega \in [0;\, \Omega_0]$. Using conventional math methods, the upper boundary for Ω_0 for revenue receipt delays is determined by the equation $\Omega_0 = \left[r_M + \sqrt{r_M^2 + 2\gamma}\right]/\gamma$, where $\gamma = 4\vec{D} \cdot (\overrightarrow{C_h} + r_M \cdot d_P \cdot \overrightarrow{C_P}/C_0$. Accordingly, the following constraint for the permitted revenue delay for the model in question should be considered:

$$0 \leq \Omega \leq \Omega_0. \quad (4)$$

When optimising supplies in case of deferrals of outgoing order payments and delays in incoming revenues, the constraint (4) mentioned above can be used *a priori*.

5. Assessment of Supply Chain Working Capital Profitability

The specified interest rate shall be estimated in the format of the traditional EOQ-model without TVM. This approach was used in [1, 2] and is used here to ensure that the results found for the profitability of the supply chain can be guaranteed, regardless of whether the decision-maker needs to take TVM into account or not. Conversely, the formula (2) is implemented with $r_M = 0$. If we consider the traditional format EOQ-model, then reorder interval is determined by the expression $\sqrt{2C_0/(\vec{D} \cdot \vec{C}_h)}$. In this case, the size of *i*-orders is defined by equality $q_{i0} = D_i \cdot \sqrt{2C_0/(\vec{D} \cdot \vec{C}_h)}$. The number of deliveries for the year is determined by the expression $\sqrt{\vec{D} \cdot \vec{\mathrm{C}}_h/2C_0}$.

Order payment, storage costs and the required deductions from profits are to be covered using the proceeds. In this situation at the beginning of the reordering period, we have costs of $L_{M(first)}$ (per year) that will be $L_{M(first)} = C_0$. For anticipated profits $Y_{(first)}$ from one delivery (without TVM) we obtain:

$$Y_{(first)} = \sqrt{\frac{2C_0}{\vec{D} \cdot \vec{C}_h}} \cdot \left[\vec{D} \cdot (\vec{P}_P - \vec{L}_P)\right] - 2C_0. \quad (5)$$

$\vec{P}_P$ is the vector of values for the units of *i*-goods supplied with coordinates $\vec{P}_P = (P_{P1}, P_{P2}, \ldots, P_{Pn})$. Vector $\vec{L}_P$ is used for the deductions from the profit per unit of *i*-goods: $\vec{L}_P = (L_{P1}, L_{P2}, \ldots, L_{Pn})$.

For the annual value of the Y profit the formula (5) is used, and thus the following expression:

$$Y = \left[\vec{D} \cdot (\vec{P}_P - \vec{L}_P)\right] - \sqrt{2C_0 \cdot \vec{D} \cdot \vec{C}_h}. \quad (6)$$

At the beginning of each reorder period, the costs $L_{M(first)} = C_0$ are needed, and therefore equation (6) allows us to determine the profitability of supply chain working capital r_M. This indicator is determined by formula $r_M = Y/L_{M(first)}$. After simple transformations we find:

$$r_M = \frac{\vec{D} \cdot (\vec{P}_P - \vec{L}_P)}{C_0} - \sqrt{\frac{2 \cdot \vec{D} \cdot \vec{C}_h}{C_0}}. \quad (7)$$

6. Order Payment Deferral Restrictions

The new modification of EOQ-formula can only be used when order payments can be covered by revenues. Therefore, conditions are now imposed on the duration of the delay $\varDelta_P$ which allows for the order payment using revenues gained during the corresponding reorder period. Therefore, exactly $q_{i\,opt}$ units of each i-product is sold during the time between deliveries T_{opt}. EOQ-model format assumes constant demand for goods. So revenues $V(t)$ considered at moment t (as a function of the variable *t)* increases linearly. The analytical expression for $V(t)$:

$$V(t) = \begin{cases} 0 \; at \; t \in [0, \Omega]; \\ (t - \Omega) \cdot \left[\vec{D} \cdot \left(\vec{C}_P + \vec{P}_P\right)\right] \; at \;\; t \in [\Omega, 1 + \Omega]. \end{cases}$$

In the simulation of such supply, a restriction can be applied to the analysis so that the costs of the first order are paid using the proceeds. This opportunity is applied automatically to subsequent orders as moments of delivery are regarded as regeneration points for the simulated processes. The value of the payment for the first order is $\vec{D} \cdot \vec{C}_P \cdot T_{opt}$. With constant demand, it takes time $t = \Omega + T_{opt} \cdot \vec{D} \cdot \frac{\vec{C}_P}{[\vec{D} \cdot (\vec{C}_P + \vec{P}_P)]}$ to obtain such revenue. Now let`s set a limit on $\varDelta_P$, so that a payment could be covered by the proceeds. $\varDelta_P$ should be such that the revenue has time to increase to the value of $\vec{D} \cdot \vec{C}_P \cdot T_{opt}$:

$$\varDelta_P \geq \Omega + T_{opt} \cdot \vec{D} \cdot \vec{C}_P / [\vec{D} \cdot (\vec{C}_P + \vec{P}_P)]. \quad (8)$$

Condition (8) is required so that the delay allowed the use of revenue for order payment.

It is necessary to account for payments that can be made with a delay of Ω relative to the middle of the reorder interval (storage costs and deductions for business support).

In general, such a study should be carried out considering the different possible situations related to the comparison allowed in the format of duration models for specific periods of time, such as $\varDelta_P$, Ω, $\frac{T_{opt}}{2}$ and T_{opt}. Here, we restrict ourselves to considering only one, but practical situation, when, we can a priori assume that inequation $\vec{D} \cdot \frac{\vec{C}_P}{[\vec{D} \cdot (\vec{C}_P + \vec{P}_P)]} \geq \frac{1}{2}$ holds. Accordingly, the fulfilment of inequation (8) can be correlated with the fact that the time of payment for storage costs and business support deductions (it is the time point $T_{opt} + \Omega$ in the first reorder interval) will precede the time of payment of the cost of the delivered order (that is, at time $\varDelta_P$), because in this situation, the inequation $\varDelta_P \geq \frac{T_{opt}}{2} + \Omega$ will hold. For this situation, in addition to condition (8), it is necessary to require that:

A. The revenue at the time of $\frac{T_{opt}}{2} + \Omega$ (previous payment of the order) turned out to be no less than $T_{opt} \cdot \vec{D} \cdot [\frac{T_{opt}\vec{C}_h}{2} + \vec{L}_P]$ so as to pay the indicated costs of storage and business support deductions using the revenue, which leads to inequation $V\left(\frac{T_{opt}}{2} + \Omega\right) \geq T_{opt} \cdot \vec{D} \cdot [\frac{T_{opt}\vec{C}_h}{2} + \vec{L}_P]]$, or to inequation $\vec{D} \cdot \left(\vec{C}_P + \vec{P}_P - T_{opt}\vec{C}_h - 2L_P\right) \geq 0$;
B. After a specified point in time $\varDelta_P$ payments should be such as to allow payment of the order (it is now be necessary to consider the previous fulfilled payments with value of $T_{opt} \cdot \vec{D} \cdot [T_{opt}\vec{C}_h/2 + \vec{L}_P]$), which leads to inequation $V(\varDelta_P) \geq T_{opt} \cdot \vec{D} \cdot [\vec{C}_P + \frac{T_{opt}\vec{C}_h}{2} + \vec{L}_P]$, or to inequation $\varDelta_P \geq \Omega + T_{opt} \cdot \vec{D} \cdot \frac{\left[\vec{C}_P + \frac{T_{opt}\vec{C}_h}{2} + \vec{L}_P\right]}{[\vec{D} \cdot (\vec{C}_P + \vec{P}_P)]}$.

In this case, instead of (8) a more general system of inequations is obtained (9):

$$\begin{cases} \vec{D} \cdot (\vec{C}_P + \vec{P}_P - T_{opt}\vec{C}_h - 2L_P) \geq 0; \\ \varDelta_P \geq \Omega + T_{opt} \cdot \vec{D} \cdot \frac{[\vec{C}_P + T_{opt}\vec{C}_h/2 + \vec{L}_P]}{[\vec{D} \cdot (\vec{C}_P + \vec{P}_P)]}. \end{cases} \quad (9)$$

The next opportunity to identify the EOQ-model of this type presents itself by combining conditions (9) and (4) into one system of inequations (subject to the required limitation $T \geq 2\Omega$) at $T_{opt} = T_0$. This will ensure the implementation of a priori established requirements, both for the permissible delay in payment for goods and the possibility of drawing order payments from revenue. These conditions are:

$$\begin{cases} \vec{D} * (\vec{C}_P + \vec{P}_P - T_0\vec{C}_h - 2L_P) \geq 0 \\ \varDelta_P \geq \Omega + T_0 * \vec{D} * \frac{\left[\vec{C}_P + \frac{T_0\vec{C}_h}{2} + \vec{L}_P\right]}{[\vec{D} * (\vec{C}_P + \vec{P}_P)]}; \\ 0 \leq \Omega \leq \Omega_0. \end{cases} \quad (10)$$

Thus, implementation of the system of inequations (10) is a necessary and sufficient condition whereby aforementioned formulas can be used to determine the parameters of the optimal supply strategy in the inventory management model considered here.

Let us turn to the analysis of situations when optimization of supplies (in the format of such inventory management models) involves cargo capacity factor of the vehicle.

7. The Specifics of Factoring in Vehicle Cargo Capacity

In the format of the multiproduct EOQ-model considered here, the following additional indicator is used:

q_{mi} $(i = 1, 2, \ldots, n)$ - the maximum number of i-goods that can be loaded in the vehicle.

Accounting for cargo capacity in inventory optimization problem (1) necessitates formalization of additional constraint.

To perform deliveries with a reorder interval T, the specified cargo capacity of the vehicle should not preclude loading of the following number of units of goods: $TD_1 + TD_2 + \cdots + TD_n$. The vector sum $T \cdot \vec{D} = (TD_1;\ TD_2;\ \ldots;\ TD_n)$ is used where i-component TD_i characterises the number of i-good units to be considered when conducting supplies using the corresponding vehicle.

The following additional concepts should be introduced to implement the constraint on the variable T:

- Analogue of the "volume" of i-product unit delivered (v_i designated and defined by $v_i = 1/q_{mi}$) when the volume of the vehicle is specified as '1'.
- Special index I_i, which is defined by $I_i = D_i \cdot v_i$ or equality $I_i = D_i/q_{mi}$.
- Total annual volume (V_i) of i-items supplied, which is $V_i = D_i \cdot v_i = D_i/q_{mi} = I_i$.
- The total annual volume (V) of supplies for the whole range of goods, which is $V = \Sigma V_i = \Sigma I_i$.
- Relevant parts w_i (where $\Sigma w_i = 1$) of the previously established vehicle cargo capacity that are allocated to corresponding i-goods supplied, as defined by equation $w_i = I_i/\Sigma I_i$.

Then we can proceed as follows in the analysis of models considering vehicle cargo factor. The limitation on the reorder interval duration T can be represented as an inequation:

$$T \leq \Delta, \text{ where } \Delta = 1/(\Sigma I_i). \tag{11}$$

Here Δ denotes the maximum duration of the time interval between deliveries where vehicle cargo capacity (a set of indicators of q_{mi} type) does not prevent loading the corresponding cargo lot.

Thus, for the generalised model of supplies, when further consideration is required for cargo capacity factor, it is necessary to consider limitations like (11) in the optimisation problem (1). This restriction should be imposed on the variable T. The time interval between deliveries, hereinafter denoted as T^*_{0M}, results in:

$$T^*_{0M} = \begin{cases} \Delta \text{ } if \text{ } \Delta < T_{opt}; \\ T_{opt} - \text{otherwise.} \end{cases} \tag{12}$$

The formal approach presented in (12) may still not conform to the format of the EOQ-model considered. For the modified EOQ-model considered here $\Omega \leq \Omega_0$ condition must be fulfilled as the model analysis assumed a priori payments for goods sold in Ω period of delay to not deter obtaining revenue within the reorder interval.

In this case, when determining the optimum duration of the time interval T^*_{0M} between cargo deliveries formulas (13) are derived from ((3 and (12)) and one of them is utilized for condition $\Omega \leq \Omega_0$ to hold a prior:

$$T^*_{0M} = \begin{cases} \Delta \text{ } if \text{ } \Delta < T_0; \\ T_0 \text{ } if \text{ } T_0 \leq \Delta. \end{cases} \tag{13}$$

Sets of formulas (2), (3), (11) and (13) presented above provide a solution to the problem set in this article for determining the optimum reorder interval for inventory management solutions involving all the factors previously mentioned. In this case, the optimal size of i-order, as noted above, for such delivery will be determined by the formula $q_i = D_i \cdot T^*_{0M}$.

8. Numerical Illustration of Simulation Results

To illustrate the corresponding optimisation procedures, a simplified model with two types of products $(i = 1, 2)$ with equal indicators is taken for each one:

- $D_i = 600$ (annual demand in units of i-products);
- $C_{Pi} = 12{,}000$ (RUB – cost of i-product per unit);
- $P_{Pi} = 3{,}000$ (RUB - profit from sale of i-product per unit);
- $L_{Pi} = 1{,}000$ (RUB - required deductions from profits per i-product unit);
- $C_0 = 80{,}000$ (RUB - the cost of one delivery);
- $C_{hi} = 4.000$ (RUB – storage costs of an i-product unit per year);
- $\Omega = 0.0192$ - duration of delays in payments for goods (years), corresponding to a one-week delay;
- $\Delta_P = 1/6$ - duration of order payment delay (years), corresponding to the two-months deferral.

Determining the optimum parameters of supply chain strategy for the following situations: 1) TVM is not considered; 2) TVM is considered.

Prior to optimisation, it is possible to estimate the annual supply chain profitability for the supply chain r_M where TVM is not taken into account, using the formula (7). To apply this formula, we find the required scalar products: $\vec{D} \cdot \vec{C}_P = 14{,}400{,}000\ (RUB)$; $\vec{D} \cdot \vec{P}_P = 3{,}600{,}000\ (RUB)$; $\vec{D} \cdot \vec{L}_P = 1{,}200{,}000\ (RUB)$; $\vec{D} \cdot \vec{C_h} = 4{,}800{,}000\ (RUB)$. Using (7) we have:

$$r_M = \frac{\vec{D} \cdot (\vec{P_p} - \vec{L}_p)}{C_0} - \sqrt{\frac{2 \cdot \vec{D} \cdot \vec{C_h}}{C_0}} =$$
$$= \frac{240000}{8000} - \sqrt{\frac{2 \cdot 4800000}{80000}} =$$
$$= 19.0455$$

In this model the profitability of working capital turned out to be unexpectedly high (compared to the case without delayed order payment, where $r_M = 0.5624$, as shown below). Illustration of further calculations is provided below.

8.1 ***Supply optimisation without TVM***

Establishing the strategy parameter using traditional EOQ-formula (with no TVM):

$$T_0 = \sqrt{\frac{2C_0}{\vec{D} \cdot \vec{C}_h}} = \sqrt{2 \cdot \frac{80000}{4800000}} = 0.182574 \text{ (years)}.$$

The required limit $2\Omega \leq T_0$ holds for this model. Therefore, the optimal re-order interval will be T_0, and for the size of the order $q_{oi}^* = D_i \cdot T_0$. For the optimal strategy without TVM: $q_{oi}^* = 600 \cdot 0{,}182574 = 109.54$ (units of goods). Let us analyse the situation where recommendation on the size of the order does not require modifications due to cargo capacity factor (e.g., where inequation $q_{mi} \geq 110$ holds for q_{mi} with $i = 1, 2$).

When optimising inventory without TVM goods must be delivered in separate cargo lots with 109.54 units of each item (on average). This strategy yields 5.477 deliveries per year. In the case where delays in order payment of two months are provided a priori, the following cash flows will be observed during the re-order period.

1. Costs at the beginning of the period: $C_0 = 80{,}000$ (RUB). The order value will be covered by revenue received (feasibility of such an arrangement will be examined and illustrated below).
2. For average revenue per delivery, we have $T_0 \cdot [\vec{D} \cdot \vec{C}_P + \vec{D} \cdot \vec{P}_P] = 3{,}286{,}332$ (RUB).
3. Order payment (covered by revenue streams) requires $T_0 \cdot \vec{D} \cdot \vec{C}_P = 2{,}629{,}065.6$ (RUB) to be paid two months after delivery of the order. It amounts to 80% of revenue. The deadline for such payments ($\Delta_P = 0.1(6)$) is greater than 80% of reorder interval $T_0 = 0.182574$. With the constant demand, such postponement will be sufficient to accumulate the sum required for order payment.
4. Storage costs and risk hedging (covered by revenue streams) amount to $T_0 \cdot [\vec{D} \cdot \vec{L}_P + T_0 \cdot \vec{D} \cdot \vec{C}_h/2] = 299{,}088.637$ (RUB). Feasibility of such payments can be verified, as it has been done for the feasibility of using the revenue for order payment.
5. For the strategy considered here, the profit from the delivery of re-order during the same period amounts to 278,177.763 (RUB).

The average expected profit for the year (the sum of profits for all shipments) is $278{,}177.763/T_0 = 1{,}523{,}643.91$ (RUB). It is achieved for the invested working capital of 80,000 (RUB). Therefore, the annual working capital profitability is $r_M = 1{,}523{,}643.91/80{,}000 = 19.0455$. As we can see, such evaluation was obtained above using the formula (7).

Moreover, if there is no delay in order payment, profitability will change significantly. In this case, obviously, the costs at the beginning of the period amount to 2,709,065.6 RUB ($= 80{,}000 + 2{,}629{,}065.6$). The profit during the re-order period will remain 278,177.763 (RUB). Annual profit will remain the same as well: 1,523,643.91 (RUB). Return on working capital r_M in a situation without delays in order payment is $r_M = 1{,}523{,}643.91/2{,}709{,}065.6 = 0.5624$ (instead of the above-mentioned value of $r_M = 19.0455$ in the situation with delayed order payment).

For calculations in real situations, when you need to use the indicator of working capital profitability, the decision-maker will seek to take additional risks into account. This can significantly decrease the profitability of r_M, as this figure has been calculated for a model with a small working capital and constant demand. We further illustrate optimisation procedures for r_M based on TVM after making adjustments for the risk of delayed receipt of revenues. Let us assume that the decision-maker reckons that it is better to use $r_M = 3.(81)$ (instead of $r_M = 19.0455$ for the “perfect” case with no delays in receipt of revenues, and instead of $r_M = 0.5624$ in the absence of delayed order payment).

8.2 ***Supply optimisation involving TVM***

For further optimization of supply strategy, $r_M = 3.(81)$ is used. Whereby $(1 + r_M \cdot \Omega) = 1.07331$ For discount factor d_P in this case, a value $d_P = 1/(1 + 3(81) \cdot \Delta_P) = 1/(1 + 3.(81)/6) = 0.6(1)$ is obtained. According to formula (2):

$$\mathrm{T}_0 = \sqrt{\frac{2C_0(1+r_M \cdot \Omega)}{\vec{D}(\vec{C}_h + r_M \cdot d_P \cdot \vec{C}_P)}} =$$
$$\sqrt{\frac{2 \cdot 80000 \cdot 1{,}07331}{4800000 + 3{,}(81) \cdot 0{,}(61) \cdot 14400000}} = 0.0(6) \text{ (years)}$$

In this situation the limitation of $2\Omega \leq T_0$ holds. Therefore $T_{opt} = T_0$, and for the optimal strategy based on TVM the size of the order is:

$q_{i\ opt} = D_i \cdot T_0 = 600 \cdot 0.0(6) = 40$ (units of i-goods).

These recommendations regarding indicators T_{opt} and $q_{i\ opt}$ assume that condition (10) holds. Therefore, condition (10) is further checked. The values of parameters $\Delta_P = 0.,1(6)$, $\Omega = 0.019$ and $T_0 = 0.0(6)$ as well as the scalar products $\vec{D} \cdot \vec{C}_P = 14{,}400{,}000$ and $\vec{D} \cdot \vec{P}_P = 3{,}600{,}000$ established before should be considered, thus:

$$0.1(6) \geq 0.0192 + 0.0(6) \cdot [14{,}400{,}000 + 2{,}400{,}000 + 1{,}200{,}000]/14{,}400{,}000 - +3{,}600{,}000).$$

After simplification, it is reduced to $0.1(6) \geq 0.0858$. This inequation holds. The second condition of (10) also holds. Thus, the mentioned above optimisation procedures can be used in the form of simulated processes.

When TVM is involved in optimisation, delivery of the goods should be done in separate cargo lots for which the expected average size is 40 units of each item. During the year there is 15 such shipments. Let us consider the cash flow structure during one re-order period of $0.0(6)$ years (approximately 24.3 days).

1. Costs at the beginning of the period equal to $C_0 = 80{,}000$ (RUB); this is the working capital.
2. Expected revenue from the delivery is $T_0 \cdot [\vec{D} \cdot \vec{C}_P + \vec{D} \cdot \vec{P}_P] = 1{,}200{,}000$ (RUB). In deterministic EOQ-model format, it is fully achieved by the time $T_0 + \Omega = 0.0858(6)$, i.e., after about 31.3 days (the first 7 of which will produce no revenue due to a delay Ω in receipt of payments for the goods).
3. Payments for storage costs and risk hedging (covered by revenue streams) require a sum of $T_0 \cdot [\vec{D} \cdot \vec{L}_P + T_0 \cdot \vec{D} \cdot \vec{C}_h/2] = 90{,}666.7$ (RUB). This sum is $7.(5)\%$ of the aforementioned revenue. This sum must be paid at time $T_0/2 + \Omega = 0.0525(3)$, which is about 19.17 days from the date of delivery (or 78.9% of the duration of the re-order interval). It is evident that such a payment can be fulfilled.
4. Payments for the order (covered by revenue streams) involve costs of $T_0 \cdot \vec{D} \cdot \vec{C}_P = 960{,}000$ (RUB) two months after delivery of the order. Those costs amount to 80% of revenue per one delivery. The deadline for such payments (two months) exceeds the reorder interval ($T_0 = 0.0(6)$, which is, about 24.3 days). The revenue covers the required order payment within the required time after payment of storage costs and risk hedging.
5. Profit per period is $1{,}200{,}000 - 80{,}000 - 90{,}666.(6) - 960{,}000 = 69{,}333.(3)$ (RUB).

The average expected annual profit is $69{,}333.(3) \cdot 15 = 1{,}040{,}000$ (RUB). It is calculated as the sum of 80,000 RUB and invested into supply chain operation (as working capital). Therefore, for the supply strategy established here, the annual profitability of working capital is $r_M = 1040000/80000 = 13.0$.

Adding TVM to inventory optimisation for a model with deferred order payment leads to a decrease in working capital profitability. Indeed, it is easy to compare this result ($r_M = 13.0$) with the result calculated without TVM ($r_M = 19.0455$). The optimisation procedure involving TVM reduced the r_M index by 6.0455 (a decrease of 31.74%).

Supply optimisation procedures that take TVM into account can be replicated in cases with no delays in receipt of revenues, i.e. when $\Omega = 0$. Then it turns out that the return on working capital will decrease by 33.64% relative to traditional guidelines, not the aforementioned 31.74%. In other words, corresponding delays in receiving the proceeds can increase the profitability of working capital.

In the format of the given model, cash investments in the work of the supply chain will be relatively small. Indeed, it is because supply chain operating costs in the given model can be paid for using the proceeds from the goods delivered. In this situation, working capital does not change with an increase in the size of the order that takes place if delays in receipt of proceeds are allowed (see (2) - (3)) taking (10) into account. In cases of fixed time limits for order payment deferrals this, in turn, positively affect the profitability of working capital for an efficient supply chain.

9. Conclusion

The newly obtained modification of EOQ-formula allows logistics managers to account for the following factors during inventory optimisation:

- deferred payment arrangements negotiated in advance for the corresponding deliveries;
- time value of money in supply chain modelling;
- delays in receipt of proceeds from the goods delivered;
- deferred payments for storage costs;
- vehicle load capacity.

The specified modification of the EOQ-formula establishes prerequisite and sufficient conditions for deferred payment of the order and storage costs to be covered by the revenues.

Presented calculations allow to illustrate that if TVM is considered, payment deferrals can significantly affect the optimal strategy parameters. The model analysed above has its own specific features such as synergies for non-deferred payment models when optimising the supply chain with TVM in terms of increased return on working capital. By optimising these models, it is possible to achieve significant improvements in profitability. However, this effect does not occur in case of the deferred order payment model considered: on the contrary, there is a decrease in the return on working capital when these optimisations are carried out using TVM.

This analysis illustrated an important feature of the EOQ-models considered, particularly additional specifics related to delays in the receipt of revenue from goods sold such as the decrease in the supply chain working capital profitability when delays in the receipt of revenue from the goods in question are allowed a priori.

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Cover letter

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22th April 2024

Dear Prof. Dr. Ching li Kuan,

We wish to submit an original research article entitled "The Modified EOQ-Formula for Deferred Order Payments and Delays in Receipt of Revenue" for consideration by Journal of Hunan University Natural Sciences.

We confirm that this work is original and has not been published elsewhere, nor is it currently under consideration for publication elsewhere.

In this research paper, we report on the study devoted to the development of a new modification of EOQ-model of inventory management. It allows for the finding of an optimal order quantity, taking into account specific factors: the time value of money, the deferral of order payment, the occurrence of delays in incoming payments, the load capacity of vehicles, and the ability to use proceeds received to pay for orders. The study presents appropriate optimal supply models, their feasibility and an example of supply fulfilment. Several curious impacts were observed: accounting for the time value of money allows for increased return on working capital, and the availability / acceptability of deferred order payment on the contrary leads to a decrease in profitability.

This is significant because of a variety(number) of both academic and practical factors. In scientific literature, this area of research is barely explored. Number of existing publications take into account only some of these factors, rather than all of them, which consequently does not (allow for) yield an optimal solution. From a practical standpoint, in the ongoing time of crisis (in particular, the increase in competition with more companies entering trading markets), companies are focusing more and more on procurement optimization, inventory management and leveraging available financial resources. This publication provides a better approach for managing the product supply and use of the company's finances.

We believe that this manuscript is appropriate for publication by Journal of Hunan University Natural Sciences because it corresponds with journal's aims and scope as the journal publishes articles in all fields of natural sciences, and the managerial area of research (including logistics) is particularly relevant today, since it concerns the effectiveness of important and widespread practical inventory management processes. In addition, a comprehensive study of the previously mentioned set of aspects and factors that impact the efficiency of inventory management is implemented and presented in print for the first time. The article is a relevant and engaging study, that addresses the topical problem of supply chain optimisation in inventory management. Results of this study will significantly improve the profitability of supplies during inventory optimization, which is important for many companies. The developed methodology and possibilities for practical application will be of interest to readers of the journal, researchers and practitioners alike.

We have no conflicts of interest to disclose.

Also, as indicated on the journal's website, we are providing additional information about featured authors:

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Thank you for your consideration of this manuscript.

Sincerely,
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On behalf of all authors