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Optimization of wholesale purchase management taking into account uncertainty and risk

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Abstract: One of the indicators of the efficiency of the enterprise in the conditions of market relations is the profit for a certain period of its work. If we consider retail enterprises, then commercial profit, in particular, depends on how efficiently the working capital of the enterprise was used (including the attracted credit resources for wholesale purchases, as well as the pricing policy for the sale of goods in the retail network. A number of models and methods for optimizing wholesale purchases of a trading company have been formed, taking into account the demand for goods, restrictions on the amount of working capital used and restrictions on the storage capacity.

The paper will consider both deterministic models and situations related to non-deterministic demand and changing retail price of goods. In this situation, multi-criteria optimization methods and methods for analyzing the stability of solutions under local perturbation of the initial parameters of the model will be used. As an illustration of theoretical models and methods, examples of the construction of optimal solutions are proposed.

Keywords : supply management, multi-criteria optimization, wholesale purchases

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Introduction

The problem of optimal supply management of a trading company under conditions of uncertainty and risk has been considered in many works.

For example, the article [1, Tobarez] states, that in a globalized and competitive market, organizations should prioritize efficient processes. Material requirements planning offers different methods to adapt to demand and determine optimal lot size at a lower cost. This study aims to develop a mathematical model applicable to all material requirements variations for informed decision-making during production.

The research [2, Risto Heikkinen] focuses on using data-driven decision support in inventory management. An interactive multi-objective optimization approach is proposed to determine batch sizes based on demand forecasts. They use a Bayesian model to forecast demand based on historical sales data. Once suitable targets are identified based on the demand model, an optimization problem is formulated to determine the batch size for future time periods.

The study [3] presents an EPQ model for exponential demand, where the rate of production exceeds the rate of demand. A mathematical model is developed to determine the optimal batch size, cycle time and total cost.

The paper [4] presents an inventory management model, namely, a model for determining the economic size of the purchase taking into account the uncertainty of demand. As an efficiency criterion, the criterion of minimization of total costs is used, taking into account the costs of excess inventory and the costs of lack of inventory. The triangular distribution is considered as the distribution law of random demand, which is one of the most applicable in conditions of lack of statistical data.

Using quantitative data analysis and computer technology, research [5] aims to develop economic and mathematical models to improve logistics and supply chain management. The study focuses on the analysis of working capital in a trading company and uses mathematical modelling and operations research to find optimal solutions. The authors also present a methodology for analyzing the stability of solutions and assessing the risk of bulk buying using a software tool based on the branch and bounds method.

Formalization of the problems of finding the optimal delivery volume and timing of deliveries and their solution by analytical methods for goods of two types, namely, with a long shelf life and with a limited shelf life is given in [6,7].

The issues of optimizing the inventory management strategy of a trading company in the context of delayed receipt of revenue are considered in the paper [8]. The study determines the acceptable deferment of payment for goods, which allows the formation of necessary payments from revenue. Reducing financial costs or losses associated with last-mile freight transportation is also discussed in [9].

The book [10] is devoted to planning and modeling investment projects for the development of logistics infrastructure and supply chain management. It presents both theoretical models of investment management in logistics and practical examples of using the proposed methodology in choosing optimal production programs for an enterprise, managing wholesale purchases of trading companies, analyzing the effectiveness of warehouse infrastructure creation projects, and managing working capital when purchasing material resources in industrial logistics.

The monograph [11] examines some models of optimization of production and financial activity of the enterprise both under conditions of deterministic initial data and taking into account uncertainty and risk (in the latter case, both the volume of a certain profit and the stability of the financed production program and the risk of its profitability are taken into account). The possibility of applying models of production and economic optimization in practice is shown, the basics of credit resource management in the real sector of the economy are revealed.

The article [12] examines theoretical and practical aspects of creating a comprehensive financial risk management system when implementing investment projects as a possible way to increase the economic stability of enterprises in the real sector of the economy. The sources of economic resources and their relationship with the algorithm for making a management decision are analyzed. A definition of a risk management system as a set of actions to transform data into a set of measures is proposed.

Paper [13] studies risk-constrained wholesale pricing contracts in a supply chain consisting of a supplier and a capital-constrained retailer.

The study [14] examines the planning of a company's activities to optimize the procurement plan in the case of multi-threading, i.e. with the possibility of ordering a product from different suppliers in the conditions of non-deterministic demand and changing prices.

The multi-criteria optimization, which is used in our work, is used to solve a wide range of different problems, including supply chain planning problems. For example, a multi-objective fuzzy linear programming model for decision making to optimize production planning, material flows, and resource allocation throughout a supply chain network considered in (15) The model focuses on both cost minimization and risk minimization based on nonsymmetric triangular fuzzy numbers.

1 Static models of optimization of the wholesale purchasing case

The verbal formulation of the problem of optimizing wholesale purchases in the general case is as follows. A trading company buys at wholesale prices various types of goods, which are then sold in the retail network at prices higher than wholesale. It is necessary to purchase at wholesale prices those goods that at a given time interval will be sold in the store, maximizing the marginal income of the enterprise. Restrictions in this task are: restrictions on demand, restrictions on the volume of goods in the wholesale warehouse, restrictions on the capacity of the warehouse of a commercial enterprise, restrictions on the amount of working capital used.

The mathematical formulation of this problem is as follows:

$$\sum_{i=1}^{n} x_i \upsilon_i \gamma_i + [F - \sum_{i=1}^{n} x_i \upsilon_i \alpha_i] \rightarrow max \qquad (1)$$

$$\sum_{i=1}^{n} x_i \upsilon_i \alpha_i \leq F \qquad (2)$$

$$x_i \upsilon_i \leq \int_0^T \div \nu_i(t)dt, i = 1,2,\dots,n \qquad (3)$$

$$0 \leq x_i \leq k_i, where\ k_i = \frac{V_i}{\upsilon_i};\ x_i \in Z^+ \qquad (4)$$

In problem (1) - (4), the following notation are used:

υ_i - the volume of the minimum lot of goods i in the wholesale purchase (*i=1,2,…,n*);

x_i - the number of minimum quantities of goods purchased wholesale (*i=1,2,…,n*);

γ_i - sales price of a type i retail (*i=1,2,…,n*);

F – working capital;

V_i - volume of goods i in stock, which can be purchased wholesale (*i=1,2,…,n*);

$k_i = \frac{V_i}{\upsilon_i}$ – the number of minimum batches of wholesale purchases in stock;

$\nu_i(t)$ - intensity of demand for goods of type i at the retail price of the product γ_i;

α_i - wholesale price of goods *i* (*i=1,2,…,n*);

Z^+- set of positive integers.

Thus, the objective function (1) sets the profit from the sale of goods. The restriction (2) indicates that in the case of bulk purchases, the costs cannot exceed the amount of working capital F. The restriction (3) indicates that all wholesale goods purchased should be sold in retail for a time period (0, T). And finally, the restriction (4) indicates that the volume of wholesale purchases of goods cannot exceed the volumes currently in stock.

Problem (1) - (4) is the problem of integer linear programming and can, in particular, be solved using the branch and bound method.

Instead of restriction (3) in model (1) - (4), constraint (3.1) - (3.2) can be used, which has the form:

$$\sum_{i=1}^{n} \alpha_i x_i \upsilon_i - \sum_{i=1}^{n} \alpha_i \times \int_0^T \nu_i(t)dt \leq \theta \cdot F \quad (3.1)$$

$$0 \leq \theta \leq 1$$

The essence of the restriction (3.1) is the assumption that part of the goods may not be sold by the end of the period (0, T), but the value of these goods must not exceed the value θF.

$$\int_0^{T+\Delta T} \nu_i(t)dt \geq x_i \upsilon_i, i = 1,2,\dots,n \qquad (3.2)$$

The restriction (3.2) indicates that with some increase in the period (0, T) by ΔT all wholesale goods will be sold out.

In addition to the listed restrictions in task (1) - (4), restrictions on the volume of a warehouse for storing goods from a retailer can additionally be specified, taking into account the possibility of additional rental of warehouse space. In this situation, the problem statement will be as follows:

$$\sum_{i=1}^{n} x_i \upsilon_i \gamma_i + F - \sum_{i=1}^{n} \alpha_i x_i \upsilon_i - W_1 \Delta_1 \rightarrow max \ (5)$$

Here W_1 – capacity of additionally leased warehouse, Δ_1– rental price for 1м^2 of warehouse.

$$\sum_{i=1}^{n} x_i \upsilon_i \alpha_i + W_1 \Delta_1 \leq F \qquad (6)$$

Equation (c) is a limit on the amount of working capital used, taking into account the cost of additional warehouse rent.

$$x_i \upsilon_i \leq \int_0^T \nu_i(t)dt, i = 1,2,\dots,n \qquad (7)$$

$$0 \leq x_i \leq k_i, x_i \in Z^+, i = 1,2,\dots,n \qquad (8)$$

$$\sum_{i=1}^{n} x_i S_i \leq W + W_1, W_1 \geq 0 \qquad (9)$$

Here S_i – the area occupied by one minimum lot of product number *i*;

W – the capacity of the warehouse of the trading company.

Obviously, if no additional rental of the warehouse is planned, i.e. $W_1 = 0$, then instead of the constraint (9) it is necessary to use the constraint:

$$\sum_{i=1}^{n} x_i S_i \leq W$$

Consider the computational scheme of the branch and bound method for solving problem (5) - (9).

1. The definition of the upper bound for the optimal value of the objective function (5).

To do this, we assume that bulk purchases can be carried out in any volume, and not in batches of the minimum volume. Next, we determine the profitability of each product according to the formula

$$d_i = \frac{\gamma_i}{\alpha_i};\ (\mathrm{i} = 1,2,\dots,\mathrm{n})$$

and renumber the types of goods so that $d_1 \geq d_2 \geq \dots \geq d_n$.

We will form a continuous wholesale purchasing portfolio as follows: first, products of the first type are purchased to the maximum extent possible, products of the second type are purchased for the rest of days, etc. until either the money runs out, or all types of goods are not purchased, or one of the restrictions is not violated (6) - (9). Next, the value of the objective function (5) on the received bulk purchase portfolio is determined. The value of the objective function on this portfolio will be considered the upper estimate F_U.

2. Determination of the lower score F_L the values of the objective function (5) on the optimal solution.

As such an estimate, one can take the value of the objective function (5) on one of the feasible solutions. In particular, such a solution can be obtained from the portfolio used in determining F_H, dividing the corresponding volumes of purchases by the value of $\upsilon_i$and discarding the fractional parts of the obtained quotient from this division. The resulting portfolio will be valid for the integer problem (5) - (9) and, by calculating on it the value of the objective function (5), we obtain the lower bound F_L.

If $F_H = F_L$, then the optimal solution to problem (5) - (9) is obtained. This decision will be a valid decision formed in determining the assessment F_L. If $F_L < F_H$, then move on to the next item of the method.

3. Formation of the next acceptable wholesale purchasing portfolio with the calculation of the current upper estimates of the objective function on this solution.

The current upper estimates of the objective function are calculated each time the next minimum batch of goods purchased in bulk is included in the purchase portfolio. The calculation of the current upper bound of the objective function on the formed portfolio of wholesale purchases is made according to the following formula:

$F_H^{current}(\tilde{\upsilon}_1, \dots, \tilde{\upsilon}_n) = \sum_{i=1}^{n} \tilde{\upsilon}_i \gamma_i + F_H(V \backslash \tilde{\upsilon})$.

Here $F_H^{current}(\tilde{\upsilon}_1, \dots, \tilde{\upsilon}_n)$- the current upper bound of the objective function on the volume of goods remaining in the warehouse, after the purchase of goods in volume $\tilde{\upsilon} = (\tilde{\upsilon}_1, \dots, \tilde{\upsilon}_n)$ and provided that working capital F', used for this are equal:

$F' = F - \sum_{i=1}^{n} \tilde{\upsilon}_i \alpha_i$.

Here F'- working capital balance;

F- the initial amount of working capital;

$\sum_{i=1}^{n} \tilde{\upsilon}_i \alpha_i$ – the amount of financial resources spent on wholesale purchases of goods in quantities $(\tilde{\upsilon}_1, \dots, \tilde{\upsilon}_n)$.

If the resulting value $F_H^{current}(\tilde{\upsilon}_1, \dots, \tilde{\upsilon}_n) \leq F_L$, then further formation of the procurement portfolio is terminated and proceed to the formation of a new portfolio for wholesale purchases.

If $F_H^{current}(\tilde{\upsilon}_1, \dots, \tilde{\upsilon}_n) > F_L$, the formation of the wholesale purchase portfolio continues, i.e. the next batch of goods for wholesale purchases is selected, it is purchased and on a variety of goods purchased in bulk are calculated again $F_H^{current}(\tilde{\upsilon}_1, \dots, \tilde{\upsilon}_n)$. Analyzing each admissible solution every time, we either reject it as non-optimal, or completely form it, and the value of the objective function (5) F^* on this decision will be more than F_L. In this case, adjust the value F_L, putting it equal F^*. If new value $F_L = F_H$, then the optimal solution is found. Or there will be a solution on which the value of the objective function (1) is F^*. If $F^* < F_H$, We continue the procedure of analyzing admissible portfolios with the calculation of the current upper estimates until one of the following events occurs:

1. With the next adjustment F_L, its value becomes equal F_H.

2. All valid portfolios are calculated and $F_L < F_H$.

In the first case, the optimal solution would be that acceptable portfolio, the value of the objective function on which is F_H. In the second case the optimal portfolio will be optimal, which corresponds to the last (maximum) value F_L.

When forming the bulk purchase portfolio, the retail price of the product has a significant impact on the result. Given that increasing the retail price of a product reduces the demand for this product, the task of optimizing the bulk purchase portfolio can be formed as follows:

$$\sum_{i=1}^{n} x_i \upsilon_i \gamma_i + [F - W_1 \Delta_1 - \sum_{i=1}^{n} x_i \upsilon_i \alpha_i] \quad (5.1)$$

$$\sum_{i=1}^{n} x_i \upsilon_i \alpha_i + W_1 \Delta_1 \leq F \quad (6.1)$$

$$x_i \upsilon_i \leq \int_0^T \left(\nu_i(t) - \delta_i (\gamma_i - \gamma_i^1) \right) dt, i = 1,2, \dots, n \quad (7.1)$$

Here δ_i – coefficient showing the intensity of demand for product i with an increase in the retail price γ_i.

$$\gamma_i^1 \leq \gamma_i \leq \gamma_i^2, i = 1,2, \dots, n \quad (8.1)$$

γ_i^1 and γ_i^2 –the maximum and minimum retail price for the product i.

$$0 \leq x_i \leq k_i, x_i \in Z^+; \ i = 1,2, \dots, n; \ W_1 \geq 0 \quad (9.1)$$

$$\sum_{i=1}^{n} x_i S_i \leq W + W_1 \quad (9.2)$$

Thus, the task with variable retail price is non-linear. Its solution consists of three components: the volume of bulk purchases of goods, given by an integer vector $x = (x_1, \dots, x_n)$, retail price vector $\gamma = (\gamma_1, \dots, \gamma_n)$ and the amount of additional rental warehouse W_1. Note also that the value $W + W_1$ in the right part of the inequality (9.2) is the total area of the warehouse for storage of goods purchased in bulk.

In practice, in some cases, a commercial enterprise, while limiting wholesale purchases and storing them in a warehouse, may additionally use credit resources attracted at a given percentage. In this situation, it is necessary to understand whether it is advisable to attract a loan from the point of view of economic efficiency and if the answer is positive, then you need to figure out how to use it (buy goods, rent a warehouse, or both).

To do this, we consider two optimization problems: Task 1 is the problem (5) - (9) (without a loan) and Task 2 is the situation when a loan is attracted. The mathematical formulation of Task 2 is as follows:

$\sum_{i=1}^{n} x_i v_i (\gamma_i - \alpha_i) - W_1\Delta_1 - L(\sum_{i=1}^{n} x_i v_i \alpha_i + W_1\Delta_1 - F) \to max$ (10)

Here L – percentage of the loan in shares.

$F < \sum_{i=1}^{n} x_i v_i \alpha_i + W_1\Delta_1 \leq F + V$ (11)

Here V – maximum amount of attracted loan.

$x_i v_i \leq \int_0^T V_i(t)dt \, , i = 1,2, \ldots, n;$ (12)

$\sum_{i=1}^{n} x_i S_i \leq W + W_1$ (13)

$0 \leq x_i \leq k_i \, , \; x_i \in Z^+, W_1 \geq 0$ (14)

Thus, solving Task 1 and Task 2, we choose the option of using working capital (with or without a loan), which gives a greater value of the objective function, respectively (5) and (10).

If at a fixed loan rate it is preferable not to attract a loan, then you can calculate the maximum rate at which it is advisable to use borrowed funds. To do this, we solve the following optimization problem:

max L (15)

Here L – loan percentage rate

$\sum_{i=1}^{n} x_i v_i (\gamma_i - \alpha_i) - L(\sum_{i=1}^{n} x_i v_i \alpha_i + W_1\Delta_1 - F) \geq \sum_{i=1}^{n} x_i^* v_i (\gamma_i - \alpha_i) = W_1\Delta_1$ (16)

Here $x^* = (x_1^*, \ldots, x_n^*)$ – optimal solution of problem (5) - (9) (without credit)

$F < \sum_{i=1}^{n} x_i v_i \alpha_i + W_1\Delta_1 \leq F + V$ (17)

$x_i v_i \leq \int_0^T v_i(t)dt, i = 1,2, \ldots, n$ (18)

$\sum_{i=1}^{n} x_i S_i \leq W + W_1$ (19)

$0 \leq x_i \leq k_i; \; x_i \in Z^+; \;\; W_1 \geq 0$ (20)

2 Stability analysis in the optimal portfolio model of wholesale purchases of goods

Furthermore, we will assume that retail prices are rising along with the growth of the accumulated inflation, and also the price for the rental of a warehouse for wholesale goods purchased is growing, that is:

$\gamma_i(\xi) = \gamma_i(0) + \varphi_i(\xi)$ (21)

$\Delta_1(\xi) = \Delta_1(0) + \psi(\xi)$ (22)

Here $\gamma_i(0)$ – initial retail price for products of type *i*;

$\gamma_i(\xi)$ – retail price for the products i at the level of the accumulated inflation ξ;

$\varphi_i(\xi)$ – the increment of prices of products of type i at the level of the accumulated inflation ξ.

$\Delta_1(0)$ – initial rental price per square meter of warehouse;

$\Delta_1(\xi)$ – rental price with accumulated inflation ξ;

$\psi(\xi)$ – the increment of rental prices of 1m^2 of wholesale at the level of the accumulated inflation ξ.

We propose to consider the set of feasible solutions to problem (5) - (9). Obviously, this set is finite. We denote it as $XW_1 = \{x^1W_1^1, \ldots, x^NW_1^N\}$. Здесь $x^jW_1^j$ – this is dimension vector n+1 and $x^jW_1^j = (x_1^j, \ldots, x_n^j, W_1^j)$.

We denote by $F^j(\xi)$ the value of the objective function (5) on solution $x^jW_1^j$ at the level of the accumulated inflation ξ, that is:

$F^j(\xi) = \sum_{i=1}^{n} x_i^j v_i \gamma_i(\xi) - \sum_{i=1}^{n} x_i v_i \alpha_i - W_1^j \Delta_1(\xi)$
(23)

Calculate the derivative of the function $F^j(\xi)$

$\frac{dF^j(\xi)}{d\xi} = \sum_{i=1}^{n} \frac{d\varphi_i(\xi)}{d\xi} \cdot x_i^j v_i - \frac{d\psi(\xi)}{d\xi} W_1^j$ (24)

Obviously, if $\varphi_i(\xi)$ and $\psi(\xi)$ are linear, then the right-hand side of inequality (24) is constant and if the right-hand side is positive, then the value of the objective function (5) grows on the solution $x^jW_1^j$ as inflation rises by linear steps. In that case, if the right-hand side of (24) is negative, then the value of the objective function (5) on the solution $x^jW_1^j$ will decrease.

In view of this fact, as well as the final amount of permissible solutions of the problem (5) - (9) we can draw the following conclusions:

1. The number of solutions of any equation $F^p(\xi) = F^q(\xi)$ on the area $\xi > 0$ is no more than one due to the linearity of the functions $F^p(\xi)$ and $F^q(\xi)$.
2. With rising inflation at any finite interval $\xi \in (0, \theta)$ there are possible transitions from one optimal solution $x^pW_1^p$ to another $x^qW_1^q$ in that case, if $\frac{dF^p(\xi)}{d\xi} < \frac{dF^q(\xi)}{d\xi}$.
3. The number of such transitions on any finite interval $(0, \theta)$ is no more than N-1, where N – the number of admissible solutions to the problem (5)-(9).
4. Any finite interval of change in inflation $(0, \theta)$ can be divided into a finite number of segments in such a way that when inflation changes within the limits of this segment, the same feasible solution of problem (5) - (9) remains optimal.

In a situation if at least one of the functions $\varphi_i(\xi)$, $\psi(\xi)$ $(i = 1,2, \ldots, n)$ is nonlinear, it is obvious that the number of transition points may be greater than N-1. As an example, consider the situation when there are two valid portfolios, $F^1(\xi)$ и $F^2(\xi)$ – piecewise linear and increasing by ξ.

The graph of these functions when changing inflation in the segment $(0, \theta)$ can be as shown in Figure 1:

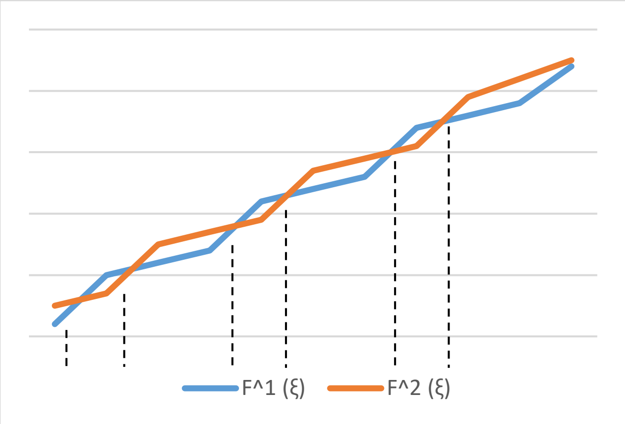

Fig.1 Trade Purchase Optimization Function Graph

Here, respectively, the levels of accumulated inflation $\xi_1, \ldots, \xi_6$ there is a transition point from the optimal solution $x^1 W_1^1$ to the solution $x^2 W_1^2$ and from the solution $x^2 W_1^2$ to the solution $x^1 W_1^1$.

3 Risk-based optimization of wholesale purchases

Below we will consider approaches for the quantitative assessment of the risk of excess purchases, the risk of lost profits and the risk of return on the wholesale purchases portfolio.

Let's consider a generalization of the problem (1) - (4), which consists in the following.

We will assume that the intensity of demand is a random function with a given distribution law

$$\nu_i(t) \begin{cases} \nu_i^1(t) - P_1 \\ \nu_i^m(t) - P_m \end{cases}$$

Here $P_j \geq 0; \sum_{e=1}^{m} P_j = 1$

Then the constraint (4) can be represented as:

$$x_i \upsilon_i \leq \int_0^T \nu_i(t)dt \quad (4.1)$$

Here $\nu_i(t) = \sum_{e=1}^{m} \nu_i^e(t) \; P_e$

If we replace the constraint (4) with the constraint (4.1) in task (1) - (4), then in this case there are two types of risks:

The risk of excess purchases, which is due to the fact that the volume of purchases is higher than the volume of sales in the store, which leads to additional costs associated with the storage of these goods in the warehouse, as well as to the loss of consumer qualities of these goods.

If the goods have a shelf life limited by a period (0, T), then the costs associated with the disposal of goods that have lost their consumer properties are also added.

The risk of loss of profits is due to the fact that the volume of demand was higher than the volume of purchases and then the company loses the marginal income from unsold goods.

Denote the mathematical expectation of total demand for products of the form i by $\overline{Pt_\iota}$, that is $\overline{Pt_\iota} = \int_0^T \nu_i(t)dt$. Denote $Pt_i^j = \int_0^T \nu_i(t)dt$. Consider examples of quantitative risk assessment of lost profits and the risk of excess purchases. If we take $\overline{Pt_\iota}$ as demand, then the real value of demand may be depending on the situation as more than $\overline{Pt_\iota}$, and less than this value. Therefore, if, in determining the optimal procurement portfolio, we obtain that $x_i \upsilon_i = Pt_i$, This is possible as a situation when the volume of wholesale products purchased exceeds demand, and a situation when real demand was greater than Pt_i. In the first case, there are losses associated with an excess of purchased products, in the second - lost profits. We define the risk of loss of profit as the expected value of lost profit due to the fact that the volume of wholesale purchases of products turned out to be less than real demand.

Formula for assessing the risk of loss of profits $R_{l.p.}$ can be defined as follows:

$$R_{l.p.} = \sum_{i=1}^{n} \beta_i \sum_{j=1}^{m} \Delta_i^j \, P_j \quad (25)$$

where β_i – is the marginal income when selling one unit of products of type i;

$\beta_i = \gamma_i - \alpha_i$;

Indicator Δ_i^j is given by the following relationship:

$$\Delta_i^j = \begin{cases} 0, if \; Pt_i^j - x_i \upsilon_i \leq 0 \\ Pt_i^j - x_i \upsilon_i, if \; Pt_i^j - x_i \upsilon_i > 0 \end{cases}$$

Where $i = 1,2, \ldots, n; j = 1,2, \ldots, m$.

Here x_i – the number of minimum quantities of goods that were purchased

The risk of excess purchases of goods $R_{e.p.}$ It estimated as the expectation of losses related to the fact that wholesale purchased goods are not sold in retail stores due to the fact that the real demand was less than the volume of purchases. Formula for quantifying $R_{e.p.}$, if we take into account only the losses associated with the loss of consumer qualities of the product, provided that their shelf life in the warehouse in the store exceeds the time interval (0, T).

$$R_{e.p.} = \sum_{i=1}^{n} d_i \sum_{j=1}^{m} \theta_i^j \, P_j \quad (26)$$

Where d_i – wholesale purchase price of products of the type i;

$$\theta_i^j = \begin{cases} 0, if \; x_i \upsilon_i - Pt_i^j \leq 0 \\ x_i \upsilon_i - Pt_i^j, if \; x_i \upsilon_i - Pt_i^j > 0 \end{cases}$$

Where $i = 1,2, \ldots, n; j = 1,2, \ldots, m$.

Next, we will consider an example of assessing the risk of excessive purchases and the risk of lost profits using formulas (25) - (26). We assume that the volume of purchases of products of the first type $v_1x_1 = 30\ units$ and of the second type $v_2x_2 = 21\ units$.

The demand for products of the first and second types is set as a random variable using the following table:

Table 1. Demand for products

Demand probabilities	The volume of demand for products of the type 1	The volume of demand for products of the type 2
1/2	30	24
1/3	27	9
1/6	36	36

Calculate assembly average (mathematical expectation) of demand for products of the first and second type.

$$\overline{Pt_1} = \sum_{j=1}^{m} Pt_1^j P_j = 30 \times \frac{1}{2} + 27 \times \frac{1}{3} + 36 \times \frac{1}{6} = 30$$

$$\overline{Pt_2} = \sum_{j=1}^{m} Pt_2^j P_j = 24 \times \frac{1}{2} + 9 \times \frac{1}{3} + 36 \times \frac{1}{6} = 21$$

Considering positive values$\overline{Pt_1}$ и $\overline{Pt_2}$, purchasing portfolio x=(30;21) is valid. We will use the following formulas to determine the risk of loss of profits and the risk of excessive purchases in conditions where $\alpha_1 = 1200\ monetary\ units$, $\alpha_2 = 1000\ monetary\ units$, $\beta_1 = 800\ monetary\ units$, $\beta_2 = 500\ monetary\ units$.

$R_{l.p.} = \sum_{i=1}^{n} \beta_i \sum_{j=1}^{m} \Delta_i^j P_j = \frac{1}{6}(36 - 30) \times 800 + \frac{1}{2}(24 - 21) \times 500 + \frac{1}{6}(36 - 31) \times 500 = 2802\ monetary\ units$

$R_{e.p.} = \sum_{i=1}^{n} d_i \sum_{j=1}^{m} \theta_i^j P_j = \frac{1}{3}(30 - 27) \times 1200 + \frac{1}{3}(21 - 9) \times 1000 = 5199\ monetary\ units$

Thus, the risk of loss of profits for the chosen example is equal to 2802 monetary units, and the risk of excess purchases - 5199 monetary units.

Taking into account the definition of the risk of loss of profits and the risk of excessive purchases, the task of minimizing the total risk can be set as $R = R_{e.p.} + R_{l.p.}$. In this case, we are talking about the choice of such a portfolio of purchases that minimizes R and maximizes the profitability of the portfolio (formula (1)).

$$R = \sum_{i=1}^{n} d_i \sum_{j=1}^{m} \theta_i^j P_j + \sum_{i=1}^{n} \beta_i \sum_{j=1}^{m} \Delta_i^j P_j \quad (26.1)$$

taking into account the limitations (2), (4), (4.1)

Considering the multicriteriality of such task, it can be used an approach based on reducing this task to a single-criterion optimization problem, for example, determining the minimum of the function (26.1) taking into account the limitations on the portfolio yield. Thus, the objective function (1) is transformed into a yield limit.

Consider a method for assessing the risk of profitability of a wholesale purchasing portfolio.

The optimization model (1) - (4) can be used to select a portfolio of wholesale purchases of goods in a situation where the future price of retail sales is fixed. In practice, due to the uncertain dynamics of a number of macroeconomic parameters, such as inflation, incomes, unemployment, world energy prices, etc., it is often impossible to accurately predict the future level of retail prices. In this case, you can use various kinds of predictive models or expert opinion. In either case, the future price can be determined at best as a random variable with a given probability distribution for its realization.

Therefore, often the future value of retail prices for goods can be determined as follows:

$$\gamma_i = \begin{bmatrix} \gamma_i^1 & P_1 \\ \dots\dots\dots & \\ \gamma_i^m & P_m \end{bmatrix},$$

$$\sum_{j=1}^{m} P_j = 1$$

$$P_j \geq 0$$

$$j = 1,2,\ldots,m$$

Here γ_i^j - value j of the retail price i;

P_j- the probability that the value of the retail price for product i will be equal to γ_i^j.

Accordingly, the mathematical expectation of the retail price $\overline{\gamma_i}$ of goods i is equal to:

$\overline{\gamma_i} = \sum_{j=1}^{m} \gamma_i^j P_j, i = 1,2,\ldots,n.$

In this situation, when forming the bulk purchase portfolio, in addition to the expected return on the portfolio, one must also consider the volatility of this yield. To quantify the volatility of profitability, you can use, following G. Markowitz, the variance of the yield of the wholesale purchases portfolio. The variance of the return on the wholesale purchasing portfolio is also an indicator of the risk of this portfolio. From this it follows that the task of forming a wholesale purchasing portfolio can be formulated as two-criteria, that is, to ensure the selection of such a portfolio that would have the maximum expected return and the minimum volatility of return.

Next, we will take into account the fact that under conditions of multi-criteria optimization, there is almost never an acceptable solution that optimizes both criteria, therefore the following technique is often used: one of the criteria is considered the main one and its maximum (minimum) is determined, and the remaining criteria are translated into restrictions. This means that when solving a single-criterion optimization problem, their values should not be less (or more) than given values. We will use this approach to formulate a risk-based model for selecting a wholesale purchase portfolio. Furthermore, we assume that the share of investment for bulk purchases of goods type i, if purchased minimum batches of goods are as follows:

$\omega_i = \frac{x_i \upsilon_i \alpha_i}{F}, i = 1,2,\dots,n.$

Here ω_i - is the share of investment in products of the type i;

υ_i - the volume of the minimum batch of wholesale purchases of goods i;

x_i - the number of minimum purchase lots of goods i;

α_i - the wholesale price of goods i;

F - the amount of working capital of a trading company.

Taking into account the above formula and using the model G. Markowitz in conditions where the main criterion is the risk of the wholesale purchases portfolio, we obtain the following optimization model:

$$\sum_{i=1}^{n} \delta_i^2 \cdot \left(\frac{x_i \upsilon_i \alpha_i}{F}\right)^2 + 2\sum_{i=1}^{n}\sum_{j>i} cov_{ij} \cdot \left(\frac{x_i \upsilon_i \alpha_i}{F}\right) \cdot \left(\frac{x_j \upsilon_j \alpha_j}{F}\right) \to min \qquad (26.2);$$

In formula (26.1):

δ_i^2 - the variance of product profitability i ($i = 1,2,\dots,n$);

cov_{ij} - the covariance of the profitability of goods i and j

($i = 1,2,\dots,n$; $j = 1,2,\dots,n$; i≠j).

$$\sum_{i=1}^{n} x_i \upsilon_i (\overline{\gamma_i} - \alpha_i) \geq D_{\text{rp}} \qquad (27).$$

Here D_{marg} – is the minimum possible yield of the wholesale purchases of goods.

$$\sum_{i=1}^{n} \frac{x_i \upsilon_i \alpha_i}{F} \leq 1 \qquad (28).$$

Ratio (28) is a consequence of restriction (2) if we divide both parts of inequality (2) into F:

$$\int_0^T v_i(t)dt \geq x_i \upsilon_i;\ i = 1,2,\dots,n\,. \qquad (29)$$

$V_i(t)$ - the intensity of retail sales of goods i at a price $\overline{\gamma_i}$.

$$0 \leq x_i \leq \frac{V_i}{\upsilon_i}, x_i \in Z^+ \qquad (30).$$

Here $\frac{V_i}{\upsilon_i}$ - is the number of minimum lots of wholesale sales of goods i in a warehouse.

Consider an example of forming an optimal wholesale portfolio procurement.

We will determine the profitability of the asset i (value d_i) by the formula:

$$d_i = \frac{\gamma_i - \alpha_i}{\alpha_i}, i = 1,2,\dots,n.$$

Consider the use of the model (26.2) - (30).

Let it be necessary to form a portfolio of bulk purchases of three types of goods - duct fans, silencers and freon coolers. The demand for these goods for the period (0, T) is 50 units, 60 units and 70 units.

The price of wholesale purchases is respectively 10 rubles per unit of the first product, 20 rubles per unit of the second product and 18 rubles per unit of the third product. The future retail price of goods γ_i is set as a random variable using the following table:

Table 2. Retail prices of goods

Probabilities	Item #1	Item #2	Item#3
1/2	17	22	20
1/3	19	23	21
1/6	18	24	25

Minimum purchase of goods $\upsilon_1 = \upsilon_2 = \upsilon_3 = 10\ items$

Amount of working capital of the enterprise F = 2300 rubles

The volume of goods available in the wholesale warehouse is equal to:

$$V_1 = 60\ items;\ V_2 = 70\ items;\ V_3 = 80\ items$$

$$\int_0^T V_1(t)dt = 50$$

$$\int_0^T V_2(t)dt = 60$$

$$\int_0^T V_3(t)dt = 70$$

Calculate the expected value of retail prices for goods i:

$\overline{\gamma_1} = 17 \cdot \frac{1}{2} + 19 \cdot \frac{1}{3} + 18 \cdot \frac{1}{6} = 8{,}5 + 6{,}3 + 3 = 17{,}8;$

$\overline{\gamma_2} = 22 \cdot \frac{1}{2} + 23 \cdot \frac{1}{3} + 24 \cdot \frac{1}{6} = 11 + 7{,}6 + 4 = 22{,}6;$

$\overline{\gamma_3} = 20 \cdot \frac{1}{2} + 21 \cdot \frac{1}{3} + 25 \cdot \frac{1}{6} = 10 + 7 + 4{,}2 = 21{,}2.$

Calculate the yield table for each type of goods:

$$d_1^1 = \frac{17 - 10}{10} = 0{,}7;$$

$$d_1^2 = \frac{19 - 10}{10} = 0{,}9;$$

$$d_1^3 = \frac{18 - 10}{10} = 0{,}8;$$

$$d_2^1 = \frac{22 - 20}{20} = 0{,}1;$$

$$d_2^2 = \frac{23-20}{20} = 0{,}15;$$
$$d_2^3 = \frac{24-20}{20} = 0{,}2;$$
$$d_3^1 = \frac{20-15}{15} = 0{,}33;$$
$$d_3^2 = \frac{21-15}{15} = 0{,}4;$$
$$d_3^3 = \frac{25-15}{15} = 0{,}66.$$

Table 3. Goods yiel

Probabilities	Item #1	Item #2	Item#3
1/2	0,7	0,1	0,33
1/3	0,9	0,15	0,4
1/6	0,8	0,2	0,66

Considering Table 3 we will make calculations, i.e. define δ_1^2, δ_2^2, δ_3^2 and cov_{12}, cov_{13}, cov_{23}.

Determine the expected value $\overline{d_i}$ of product returns ($i = 1,2,\ldots,n$)

$$\overline{d_1} = 0{,}7\cdot\frac{1}{2} + 0{,}9\cdot\frac{1}{3} + 0{,}8\cdot\frac{1}{6} = 0{,}35 + 0{,}3 + 0{,}13 = 0{,}78;$$
$$\overline{d_2} = 0{,}1\cdot\frac{1}{2} + 0{,}15\cdot\frac{1}{3} + 0{,}2\cdot\frac{1}{6} = 0{,}05 + 0{,}05 + 0{,}03 = 0{,}13;$$
$$\overline{d_3} = 0{,}33\cdot\frac{1}{2} + 0{,}4\cdot\frac{1}{3} + 0{,}66\cdot\frac{1}{6} = 0{,}165 + 0{,}133 + 0{,}11 = 0{,}41.$$

Calculating the variance of returns δ_1^2, δ_2^2, δ_3^2

$$\delta_1^2 = \sum_{l=1}^{3}\left(\overline{d_1} - d_1^l\right)^2 P_l = (0{,}78-0{,}7)^2\cdot\frac{1}{2} + (0{,}78-0{,}9)^2\cdot\frac{1}{3} + (0{,}78-0{,}8)^2\cdot\frac{1}{6} = = 0{,}064\cdot\frac{1}{2} + 0{,}0144\cdot\frac{1}{3}0{,}0004\cdot\frac{1}{6} = 0{,}032 + 0{,}005 + 0{,}00006 = 0{,}037;$$

$$\delta_2^2 = \sum_{l=1}^{3}\left(\overline{d_2} - d_2^l\right)^2 P_l = (0{,}13-0{,}1)^2\cdot\frac{1}{2} + (0{,}13-0{,}15)^2\cdot\frac{1}{3} + (0{,}13-0{,}2)^2\cdot\frac{1}{6} = = 0{,}00045 + 0{,}00013 + 0{,}0008 = 0{,}00138;$$

$$\delta_3^2 = \sum_{l=1}^{3}\left(\overline{d_3} - d_3^l\right)^2 P_l = (0{,}41-0{,}33)^2\cdot\frac{1}{2} + (0{,}41-0{,}4)^2\cdot\frac{1}{3} + (0{,}41-0{,}66)^2\cdot\frac{1}{6} = = 0{,}32 + 0{,}00003 + 0{,}014 = 0{,}334.$$

Defining the values and cov_{12}, cov_{13}, cov_{23}.

$$cov_{12} = \sum_{l=1}^{3}\left(\overline{d_1} - d_1^l\right)\cdot\left(\overline{d_2} - d_2^l\right)\cdot P_l = = (0{,}78-0{,}7)(0{,}13-0{,}1)\cdot\frac{1}{2} + (0{,}78-0{,}9)(0{,}13-0{,}15)\cdot\frac{1}{3} + (0{,}78-0{,}8)(0{,}13-0{,}2)\cdot\frac{1}{6} = 0{,}012 + 0{,}0006 + 0{,}0002 = 0{,}0128;$$

$$cov_{13} = \sum_{l=1}^{3}\left(\overline{d_1} - d_1^l\right)\cdot\left(\overline{d_3} - d_3^l\right)\cdot P_l = = (0{,}78-0{,}7)(0{,}41-0{,}33)\cdot\frac{1}{2} + (0{,}78-0{,}9)(0{,}41-0{,}4)\cdot\frac{1}{3} + (0{,}78-0{,}8)(0{,}41-0{,}66)\cdot\frac{1}{6} = 0{,}32 - 0{,}00004 + 0{,}0002 = 0{,}32;$$

$$cov_{23} = \sum_{l=1}^{3}\left(\overline{d_2} - d_2^l\right)\cdot\left(\overline{d_3} - d_3^l\right)\cdot P_l = = (0{,}13-0{,}1)(0{,}41-0{,}33)\cdot\frac{1}{2} + (0{,}13-0{,}15)(0{,}41-0{,}4)\cdot\frac{1}{3} + (0{,}13-0{,}2)(0{,}41-0{,}66)\cdot\frac{1}{6} = 0{,}0012 - ,00007 + 0{,}018 = 0{,}019.$$

Forming numerically model (26.1) - (30) for the considered example:

$$0{,}037\cdot\left(\frac{x_1\cdot 10\cdot 10}{2300}\right)^2 + 0{,}00138\cdot\left(\frac{x_2\cdot 10\cdot 20}{2300}\right)^2 + 0{,}334\cdot\left(\frac{x_3\cdot 10\cdot 15}{2300}\right)^2 + 2\cdot\left(0{,}0128\frac{x_1\cdot 10\cdot 10}{2300}\cdot\frac{x_2\cdot 10\cdot 20}{2300} + 0{,}32\frac{x_1\cdot 10\cdot 10}{2300}\cdot\frac{x_3\cdot 10\cdot 15}{2300} + 0{,}019\frac{x_2\cdot 20\cdot 10}{2300}\cdot\frac{x_3\cdot 10\cdot 15}{2300}\right) \to min \text{ (13.1)};$$

Defining the value of the marginal portfolio yield as $D_{marg} = 2500$.

$$x_1\cdot 10\cdot(17{,}8-10) + x_2\cdot 10\cdot(22{,}6-20) +$$

$x_3 \cdot 10 \cdot (21{,}2 - 15) \geq 2500;$ (14.1)

$\frac{x_1 \cdot 10 \cdot 10}{2300} + \frac{x_2 \cdot 10 \cdot 20}{2300} + \frac{x_3 \cdot 10 \cdot 15}{2300} \leq 1;$ (15.1)

$x_1 \cdot 10 \leq 50 \ => x_1 \leq 1;$ (16.1)

$x_2 \cdot 10 \leq 60 \ => x_2 \leq 6;$

$x_3 \cdot 10 \leq 70 \ => x_3 \leq 7;$

$0 \leq x_1 \leq \frac{60}{10} = 6;$ (17.1)

$0 \leq x_2 \leq \frac{70}{10} = 7;$

$0 \leq x_3 \leq \frac{80}{10} = 8; \quad x_i \in Z^+$

Here Z^+- the set of positive integers.

Combining the restrictions (16.1) and (17.1), we can write down the restrictions on the number of purchased goods in the following form

$x_1 \leq 5;\ x_2 \leq 6;\ x_3 \leq 7;\ x_i \in Z^+;\ i = 1,2,3.$

Thus, task (13.1) - (15.1), (16.2) defines the minimum risk wholesale purchase portfolio with a limit on its expected return for a parsed example.

4 Dynamic model for Optimizing the Wholesale Purchasing Portfolio

This model assumes that retail prices for goods may change during its implementation. A mathematical model for optimizing a wholesale purchasing portfolio can be formed as follows:

$\sum_{i=1}^{n} \int_0^T (\gamma_i(t) \cdot \theta_i(t, \gamma_i(t)) dt + [F_2 - \sum_{i=1}^{n} \alpha_i\, x_i v_i - W_1 \Delta_1] \longrightarrow max$ (31)

$W_1 \geq 0; 0 \leq x_i \leq k_i;\ x_i \in Z^+;\ i = 1,2, \dots, n$ (32)

$\gamma_i^1 \leq \gamma_i \leq \gamma_i^2;\ i = 1,2, \dots, n$ (33)

$\sum_{i=1}^{n} x_i S_i \leq W + W_1$ (34)

$W_1 \Delta_1 + \sum_{i=1}^{n} x_i v_i \alpha_i \leq F_2$ (35)

$\int_0^T \theta_i(t, \gamma_i(t)) dt = x_i v_i;\ i = 1,2, \dots, n$ (36)

$\int_0^t \theta_i(t', \gamma_i(t')) dt' \leq \int_0^t \nu_i(t', \gamma_i(t')) dt'; V t \epsilon (0, T)$ (37)

$\gamma_i(t) \geq 0;\ i = 1,2, \dots, n$ (37.1)

In the model (31) - (37), the following notation was used:

$\gamma_i(t)$ –the retail price of the item i at the time $t \epsilon (0, T)$

$\theta_i(t, \gamma_i(t))$ – the intensity of sales of the item i at the time t at the retail price $\gamma_i(t)$

F_2 – the amount of working capital of the trading company

$\gamma_i^1; \gamma_i^2$ – the maximum and minimum sales price of the product i in the interval (0,T)

$V_i(t', \gamma_i(t'))$ – the intensity of demand for product i at the time at the price$t' \epsilon (0, T)$ when the price is $\gamma_i(t')$

x_i – the number of goods parities

Solution of the problem (31) - (37) will be: the procurement vector $x = (x_1, \dots, x_n)$, the vector function $\gamma(t) = (\gamma_1(t), \dots, \gamma_n(t))$, specifying the retail price goods at each time point $t = (0, T)$, additionally rentable warehouse area W_1, and vector function $\theta(t, \gamma(t)) = (\theta_1(t, \gamma_1(t)),$ $(\theta_2(t, \gamma_2(t)), \dots, (\theta_n(t, \gamma_n(t))$, defining the intensity of sales of goods. In the general formulation, problem (31) - (37.1) is the optimal control problem.

If the vector function$\gamma(t) = (\gamma_1(t), \dots, \gamma_n(t))$ and the vector function $\theta(t, \gamma(t))$ are defined on the set of piecewise constant functions, then problem (31) - (37.1) can be reduced to the problem of quadratic optimization.

5 Conclusion

In this study, a number of deterministic and non-deterministic models for optimizing wholesale purchases of a trading enterprise were presented, taking into account the demand for goods and various restrictions, such as restrictions on the amount of working capital used and restrictions on warehouse capacity.

Situations of non-deterministic demand and prices, as well as a multi-criteria approach, are relevant issues in optimizing the management of operating funds in supply chains.

Theoretical models were tested on calculation examples.

References

1. T.D. TOBAREZ, M.M. MIERAS, R.R. PALMA et al. Theoretical relationship between the cluster size of orders in the materials requirement planning. International Journal of Logistics Systems and Management 2023 46:1, 27-46. DOI: 10.1504/IJLSM.2023.133520

2. R. HEIKKINEN, J. SIPILA, V. OJALEHTO et al. Flexible data driven inventory management with interactive multi-objective lot size optimization. International Journal of Logistics Systems and Management 2023 46:2, 206-235. https://doi.org/10.1504/IJLSM.2023.134404

3. R.P. TRIPATHI, M. KAUR. Optimal ordering policies in economic production quantity model with exponential time - dependent demand and diminishing release strategy. International Journal of Logistics Systems and Management 2023 45:3, 317-329. DOI: 10.1504/IJLSM.2023.133004.

4. O. KOSORUKOV, S. MASLOV, O. SVIRIDOVA. Algorithm for Determining the Economic Purchase Size in the Conditions of the Demand Volatility. May 2020, https://doi.org/10.5373/jardcs/v12sp5/20201866.

5. A. MISHCHENKO, O. KOSORUKOV, T. APALKOVA et al. Management models and methods of current assets in trading company. 12.2023. 112-128. DOI: 10.36871/2618-9976.2023.12.0011

6. O. KOSORUKOV, I. ILIN, O. SVIRIDOVA. Models for Optimisation of Delivery Timing and Volumes under Uncertainty Encompassing Penalty and Customer Attrition Risks. In International Journal of Recent Technology and Engineering (IJRTE) (Vol. 8, Issue 4, pp. 11539–11545). Blue Eyes Intelligence Engineering and Sciences Engineering and Sciences Publication - BEIESP. https://doi.org/10.35940/ijrte.d4407.118419

7. O. KOSORUKOV, S. MASLOV, O. SVIRIDOVA et al. Models for Optimizing the Supply Volume in Conditions of Uncertainty in Demand, Taking into Account the Risks of Accrual of Penalties, Loss of Customers and Additional Costs. *Lecture Notes in Networks and Systems*, 2022, 387, страницы 249–259, DOI: 10.1007/978-3-030-93872-7_21

8. G. L. BRODETSKIY, V. GERAMI, I.G. SHIDLOVSKII. The Modified EOQ-Formula for Deferred Order Payments and Delays in Receipt of Revenue. Journal of Hunan University (Natural Sciences), 2024, 51(2). https://doi.org/10.55463/issn.1674-2974.50.9.31

9. G. L. BRODETSKIY, V. GERAMI, I.G. SHIDLOVSKII. Improving the Efficiency of Logistics Activities at the Last Mile by Optimizing the Queue of Order Execution. Conference: XXIII Yasin International Academic Conference on Economic and Social, Development, National Research University Higher School of Economics, April 2022, https://www.researchgate.net/publication/359770959.

10. A. MISHCHENKO. Methods and models of investment management in logistics. M.: INFRA-M, 2020. https://infra-m.ru/catalog/upravlenie_menedzhment/metody_i_modeli_upravleniya_investitsiyami_v_logistike_uchebnoe_posobie_2/

11. A. MISHCHENKO, E. VINOGRADOVA. Optimization models of enterprise financial resource management. M.: INFRA-M, 2015. https://publications.hse.ru/books/175064840

12. O. PANFILOVA. Financial Risk Management System for the Implementation of Investment Projects. Izvestia Sankt-Peterburgskogo gosudarstvennogo ekonomicheskogo universiteta, 2012, issue 4, 103-106.

13. HONGLIN YANG, WENYAN ZHUO, LUSHENG SHAO et al. Mean-variance analysis of wholesale price contracts with a capital-constrained retailer: Trade credit financing vs. bank credit financing, European Journal of Operational Research, Volume 294, Issue 2, 2021, Pages 525-542, ISSN 0377-2217, https://doi.org/10.1016/j.ejor.2021.01.042.

14. S. THEVENIN, O. BEN-AMMAR, N. BRAHIMI Robust optimization approaches for purchase planning with supplier selection under lead time uncertainty, European Journal of Operational Research, Volume 303, Issue 3, 2022, Pages 1199-1215, ISSN 0377-2217, https://doi.org/10.1016/j.ejor.2022.03.029.

15. N. SUTTHIBUTR, K. HIRAISHI, N. CHIADAMRONG. A fuzzy multi-criteria decision-making for optimizing supply chain aggregate production planning based on cost reduction and risk mitigation, Journal of Open Innovation: Technology, Market, and Complexity, Volume 10, Issue 4, 2024, 100377, ISSN 2199-8531, https://doi.org/10.1016/j.joitmc.2024.100377.