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Efficiency Estimates of Deliveries Using Any Number of Vehicles for an EOQ Model Considering Delays in Receiving Revenue and Cargo Capacity

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Abstract: This article presents a novel approach to evaluating the profitability of working capital in supply chains by applying specialized modifications to the EOQ (Economic Order Quantity) inventory management model framework. The research addresses the critical issue of optimizing inventory management and determining the duration of the delay in payment to the supplier factoring in the TVM (Time Value of Money), transportation logistics constraints, and allowable delays in receiving revenue from fulfilled orders. Specifically, this study focuses on scenarios where payments for orders can be covered by the revenue generated from delivering those orders, effectively making order and delivery costs part of the working capital. The objective is to provide an approach within the EOQ model framework that enables the determination of profitability while optimizing the order size considering the vehicle capacity, delivery cost discounts, the use of multiple vehicles, and delays in receiving revenue. The approach involves deriving interest rate estimates that reflect the profitability of the supply chain under mentioned conditions. The results show that, under certain conditions, using multiple vehicles can enhance profitability, but only when discounts exceed a threshold value and when delays in revenue receipt are properly accounted for. By incorporating logistical constraints and financial timing factors, proposed procedures for assessing interest rates improves the accuracy of inventory management strategies, contributing to more efficient supply chain operations.

Keywords: Economic Order Quantity (EOQ), delays in receiving revenue, vehicle capacity, multi-vehicle delivery, working capital profitability, transportation cost discounts.

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Introduction

Various approaches are currently employed in inventory optimization, including the consideration of the time value of money (TVM). Incorporating TVM can be conveniently achieved using simple interest schemes, as proposed in [1]. This choice stems from a specific characteristic of Economic Order Quantity (EOQ) models: the profit obtained cannot be used to increase the predetermined annual delivery volume (e.g., to generate even greater profit). Indeed, the exogenously specified demand is an inherent attribute of the EOQ model. Therefore, when considering TVM, we should refer to models where, in financial analysis terms, "interest is not compounded," aligning precisely with the characteristics of simple interest.

To model these systems using modified EOQ formulas that incorporate TVM, we need to estimate the corresponding interest rate. The specified interest rate should reflect the efficiency of converting outgoing cash flows into profit within the modeled supply chain. This aligns with modeling procedures developed in financial analysis and financial management theory. In practice, estimating this interest rate is best done by consulting directly with the decision-maker (DM), who has an accurate understanding of all cash flows. However, it may sometimes be necessary for managers to estimate this rate themselves.

To assist managers in implementing and illustrating these procedures, we have analytically derived the required interest rate estimates in this article. This facilitates their use in various modeling and optimization approaches, both simulation and analytical. Our analysis is conducted specifically for situations of practical importance.

Accordingly, we develop the estimation procedures considering the specifics of the single-item EOQ inventory management model discussed here. Specifically, we address important practical factors:

a) An a priori allowable delay in revenue receipt;

b) The consideration of vehicle capacity;

c) The number of vehicles involved in deliveries and the specifics of the working capital structure.

To ensure the results are both interesting and valuable for businesses, our research focuses on an important class of models corresponding to a particular aspect of supply chain efficiency [6]. Specifically, we consider models where it is assumed that required payments during the reorder interval are made from the revenue generated by delivering the order. These payments exclude expenses such as the cost of the order and its delivery, which are covered by working capital. Consequently, the working capital considered is the sum of the order cost and delivery cost. We specify the conditions that allow these models to be identified, enabling managers to apply the obtained interest rate estimates in practical scenarios.

Therefore, the purpose of this article is to provide managers with analytical estimates of the interest rate characterizing the efficiency of working capital utilization in inventory optimization. We consider the specifics of delivering orders using any number of vehicles within the framework of the special EOQ model. Our findings also enable consideration of special scenarios in modeling, including aspects related to the organization of order deliveries in inventory optimization: accounting for discounts on delivery costs when increasing the number of vehicles, considering allowable delays in revenue receipt, recognizing the potential to cover required payments from revenue, and factoring in load capacity.

1. Theoretical Background

The integration of the TVM is illustrated by the work [13], highlighting how inflationary conditions impact inventory decisions. The paper [18] introduces an EOQ model with constant demand, integrating TVM alongside factors like deterioration of goods in storage and delayed payments.

Considering TVM as continuously compounded interest is discussed in several studies. For example, [8] utilizes discounted cash flow analysis for an EOQ model where the supplier offers various trade credits. A generalization incorporating discounted cash flow and two levels of trade credit is presented in [19]. The study [11] illustrates an inventory management model addressing uncertainties inherent in demand forecasting. Similarly, the study [17] presents a simulation model for optimizing inventory replenishment of perishable goods with uncertain demand, aiming to maximize expected profit flow by determining economically feasible order volumes and timing. Additionally, [14] proposes an EOQ model considering stock-dependent demand and two levels of trade credit along with TVM.

In addition, an important issue is the inclusion of vehicle capacity and shipping discounts in EOQ calculations. Burwell et al. [7] developed an economic lot size model that incorporates price-dependent demand along with quantity and freight discounts. The study [16] developed algorithms to determine the optimal order quantities when vendors offer products in various container sizes with larger discounts on larger sizes. Research [12] introduced a complex algorithm that simultaneously applies all-units quantity discounts on purchasing price and freight cost. Article [15] introduced a marginal-cost solution procedure for single-period inventory models with quantity discounts. [20] presented an analytical procedure for finding the order quantity that minimizes total purchase costs by reflecting both transportation economies and quantity discounts. Article [22] developed decomposition rules for breaking down incremental quantity and weight discounts into other discount scenarios.

2. Attributes of Inventory Management Model Involving TVM

Note that the index i is used to denote the type of vehicles used for order deliveries. This feature may prove useful if, during model optimization, it becomes necessary to select the specific type of vehicles for delivering orders.

Indication of the notations used:

- D – annual consumption for the product;
- C_P – the cost of the product per unit;
- P_P – the profit from the sale of the product per unit (required for considering the TVM);
- L_P – charges per unit of the product required to maintain the business (required for considering the TVM);
- C_h – annual costs of storage per unit of the product;
- Ω – the duration of the allowable delay in receiving revenue from sold products (years);
- C_{0i} – the cost of one delivery by the type i vehicle (these are expenses that do not depend on the size of the order and therefore cannot be attributed to the cost per unit of products);
- $d_i^{(k)}$ – the discount on the cost of the one delivery if it is carried out using k number of type i vehicles simultaneously;
- q_{mi} – maximum allowable order size: capacity of the type i vehicle (units of product);
- q – the size of the order, an optimizable value; this indicator is specified considering the delivery structure: when optimally using k vehicles of the type i for delivering goods considering the TVM, the order size is denoted as $q_i^{(k)}$;
- T – the period between deliveries or the reorder interval (linked to the order size by the equality $T = q/D$), also an optimizable value (years); this indicator is also specified with respect to the delivery structure: when optimally using k vehicles of the i-type for delivering goods, considering the TVM, the reorder interval for such deliveries is denoted as $T_i^{(k)}$;
- r_i – interest rate indicator characterizing the efficiency of converting the working capital required for the operation of the supply chain into profit when deliveries are carried out by one vehicle of the i-type; this indicator must be used if the TVM is considered in optimization (measured as a fraction relative to the invested working capital and defined by formulas that will be presented later);
- $r_i^{(k)}$ – similar indicator for the situation when deliveries are carried out using k vehicles of the i-type simultaneously; formulas for this indicator are also presented later in this article.

According to inventory management theory, the optimal decision regarding the organization of deliveries should minimize the total annual costs associated with order deliveries. These include the costs of storing the goods, their delivery, purchasing the goods, and other expenses.

In considering the load capacity factor, we examine situations where order deliveries are carried out using multiple vehicles simultaneously. Accordingly, we must account for the following constraint: the maximum quantity of goods in one vehicle during deliveries must not exceed a specific given value q_{mi} corresponding to the capacity of vehicles of type i.

Naturally, the decision on deliveries when optimizing them will depend on several factors. In particular, the following factors need to be considered:

1. The use of the TVM concept in optimizing delivery decisions.
2. Accounting for the vehicle capacity factor in the context of such decisions.
3. Considering the specific number of vehicles planned to be used simultaneously for order deliveries.
4. The specifics of cash flows, including the structure of working capital.

Notably, we are considering a model where inventory holding costs are accounted for based on the occupied storage spaces, as in the classical Harris-Wilson model.

Moreover, it should be noted that allowable delays in receiving revenue from sold goods will be considered in the model discussed here. Also, recall that the model format a priori assumes the possibility of making the required payments from revenue. In this case, the cost of the order (including the costs of its delivery) is considered as working capital that needs to be invested in the operation of the supply chain.

Let us emphasize once again that the interest rate estimates of concern should characterize the profitability of the modeled supply chain's operation. To ensure this—specifically in the situations considered—we will naturally use previously obtained results for the parameters of the optimal order delivery strategy within the corresponding EOQ model. In other words, special modified formulas will be used here; these are formulas for both the optimal order size and the reorder interval, presented in [1 – 5].

In this regard, let's pay attention to the following features of the procedures for optimizing deliveries within such models. When optimally using k vehicles of the i-type and considering the TVM, the total order size $q_i^{(k)}$ for such deliveries should be determined by the formula (1).

$$q_i^{(k)} = \sqrt{\frac{2D\left(1-d_i^{(k)}\right)kC_{0i}\cdot\left(1+r_i^{(k)}\Omega\right)}{C_h+r_i^{(k)}C_P}} \quad (1)$$

The optimal reorder interval $T_i^{(k)}$ for such deliveries should be determined by the formula (2).

$$T_i^{(k)} = \sqrt{\frac{2\left(1-d_i^{(k)}\right)kC_{0i}\cdot\left(1+r_i^{(k)}\Omega\right)}{D\left(C_h+r_i^{(k)}C_P\right)}} \quad (2)$$

As we can see, when using formulas (1) – (2) for optimizing the order delivery strategy, it is essential to know the estimates for the corresponding indicators of the type $r_i^{(k)}$. This is precisely what was discussed when formulating the research problem presented in this article.

Let us further clarify another feature. When considering the vehicle capacity factor, the formula (3) should be used [5][10].

$$q_i^{*(k)} = \begin{cases} q_i^{(k)}, if\ q_i^{(k)} \le k\cdot q_{mi}; \\ k\cdot q_{mi} - otherwise. \end{cases} \quad (3)$$

Similarly, adjustments should be made when determining the optimal reorder interval considering the load capacity factor.

3. Estimates for the Supply Chain Profitability Indicator

The required estimates for the value of the interest rate of concern ($r_i^{(k)}$) should account for the specific formats of possible situations that need to be correlated with the implementation of the considered model. In particular, it is necessary to consider that the optimal solution may be achieved when orders are delivered using multiple vehicles simultaneously. In such situations, discounts on the cost of these deliveries using multiple vehicles must be taken into account. Accordingly, when determining the estimates of interest for such indicators, it is convenient to immediately consider the general case of order deliveries using k vehicles. Naturally, in the situation where $k=1$, this corresponds to the traditional case in theory.

Moreover, when estimating the specified interest rate, we must also consider the following feature of such procedures. It is essential to determine whether the vehicles used in such deliveries are loaded to their maximum capacity or not.

We will present the corresponding estimates for the specified interest rate $r_i^{(k)}$, which can be used in optimizing deliveries applicable to possible situations in the analysis:

1. The optimal order size is determined by the given limitation of $k\cdot q_{mi}$ (using k i-type vehicles for deliveries).
2. The optimal order size is determined using the corresponding modification of the EOQ formula (considering the capacity is unnecessary).

The specified indicator $r_i^{(k)}$ should characterize the efficiency of converting the working capital into profit. It is necessary to consider that k vehicles of the i-type are used simultaneously for delivering goods and that the discount $d_i^{(k)}$ is provided on the cost of such delivery.

To determine the indicator $r_i^{(k)}$, we use cash flow modeling associated with the operation of the supply chain. The procedures will depend on the degree of loading of these k vehicles of the i-type. Therefore, an analysis of two special situations was conducted to account for this feature.

3.1 Full Vehicle Load ($q_i^{*(k)} = kq_{mi}$)

First, let's consider the case where the estimate for the indicator $r_i^{(k)}$ is required in a situation where the load capacity factor influences the procedures for optimizing deliveries. In this case, when using k vehicles of type i, the average number of such deliveries per year is $D/(kq_{mi})$.

We assume that the investments in the operation of the supply chain over one reorder interval represent the following: costs of paying for the order ($q_i^{*(k)}\cdot C_P$) and the costs of paying for the delivery considering possible discounts ($\left(1-d_i^{(k)}\right)kC_{0i}$).

Recall that the model format of interest here (with allowable delay in revenue receipt) assumes that over the reorder interval, the payment of holding costs $(kq_{mi})^2C_h/(2D)$ is made from the revenue. Therefore, these costs are not included in the working capital. Additionally, we must consider that the required deductions $kq_{mi}L_P$ for each reorder interval will also be made from the revenue. The conditions that need to be imposed on the parameters of the EOQ model so that these payments can be made from the revenue will be noted later. Thus, the following can be noted.

As working capital for the operation of the supply chain using k vehicles at the beginning of the period, expenditures L_I will be required. These are the expenditures for paying for the cost of the order and the cost of the delivery, which (taking into account the corresponding discount) amount to:

$$L_I = q_{mi}kC_P + \left(1-d_i^{(k)}\right)kC_{0i}. \quad (4)$$

The profit over one reorder interval Y_I for such deliveries will be equal to:

$$Y_I = kq_{mi}(P_P - L_P) - \left(1-d_i^{(k)}\right)kC_{0i} - \frac{(k\,q_{mi})^2C_h}{2D}. \quad (5)$$

Let us find the average expected annual profit Y. The modeling procedures for estimating the interest rate are implemented precisely for the traditional format of the delivery model: in these calculations, we do not take into account the TVM so that the result does not depend on the DM's choice of whether to consider TVM in optimization [1] – [5]. Thus, the indicator Y is determined by the formula $Y = Y_I D/(kq_{mi})$ (as the expected annual profits from all deliveries). Accordingly, we obtain its following value:

$$Y = D \cdot (P_P - L_P) - D\left(1 - d_i^{(k)}\right) C_{0i}/q_{mi} - k\, q_{mi} C_h/2 \,. \quad (6)$$

Now, considering the previously specified value of the working capital, we find (7), which provides the estimate for the required profitability indicator:

$$r_i^{(k)} = \frac{D(P_P - L_P) - \frac{DC_{0i}\left(1-d_i^{(k)}\right)}{q_{mi}} - kq_{mi}C_h/2}{kq_{mi}C_P + \left(1-d_i^{(k)}\right)kC_{0i}}. \quad (7)$$

As we can see, equation (7), can be used to find the interest rate characterizing the efficiency of converting cash outflows into the profit in the operation of the modeled supply chain. Naturally, such a formula can be used only applicable to the situation where each of the k vehicles of i-type in deliveries is fully loaded).

3.2 Partial Vehicle Load ($q_i^{*(k)} = q_i^{(k)}$)

In this section, we analyse a scenario where, within the optimization procedures, the load capacity factor does not influence the delivery decision. This occurs when the specified limitation on vehicle load capacity does not affect the order size during optimization.

We are now considering a situation where each of the k vehicles is loaded according to the value determined by the previously mentioned special modified EOQ formula. For this scenario, the optimal order size $q_i^{*(k)} = q_i^{(k)}$ is given by

$$q_i^{(k)} = \sqrt{2D\left(1 - d_i^{(k)}\right) kC_{0i}/C_h}. \quad (8)$$

And the average number of deliveries per year is:

$$D/q_i^{(k)} = \sqrt{C_h D/\left[2\left(1 - d_i^{(k)}\right) kC_{0i}\right]}. \quad (9)$$

As in the previous situation, we recognize that the investments in operating the supply chain comprise the following amount: costs of paying for the order ($q_i^{*(k)} C_P$) and the costs of paying for the delivery considering possible discounts $(1 - d_i^{(k)})kC_{0i}$.

Furthermore, required deductions L_P (an amount of $q_i^{(k)} L_P$ for each reorder interval) are also made from revenue. It is important to note the following characteristic of the current scenario.

As working capital for the operation of the supply chain using k vehicles at the beginning of the period, expenditures L_I will be required:

$$L_I = q_i^{(k)} C_P + \left(1 - d_i^{(k)}\right) kC_{0i}. \quad (10)$$

The profit over one reorder interval Y_I for such deliveries will be equal to:

$$Y_I = q_i^{(k)}(P_P - L_P) - 2\left(1 - d_i^{(k)}\right) kC_{0i}. \quad (11)$$

To determine the average expected annual profit Y, we use the formula $Y = Y_I D/q_i^{(k)}$. We calculate the average expected profit from all deliveries over the year:

$$Y = D \cdot (P_P - L_P) - \sqrt{2\left(1 - d_i^{(k)}\right)kC_{0i}DC_h}. \quad (12)$$

Considering the aforementioned value of working capital (L_I), we derive formula (13) for the annual profitability indicator $r_i^{(k)}$. Recall that we are examining the situation when the load capacity factor does not affect the order size:

$$r_i^{(k)} = \frac{D(P_P - L_P) - \sqrt{2\left(1-d_i^{(k)}\right)kC_{0i}DC_h}}{q_i^{(k)} C_P + \left(1-d_i^{(k)}\right)kC_{0i}}. \quad (13)$$

Formula (13) enables us to determine, for any k, the required interest rate $r_i^{(k)}$ that characterizes the efficiency of converting monetary outflows (associated with operating the supply chain) into profit. This formula is derived for models utilizing the traditional optimization approach based on the corresponding modification of the EOQ formula, applicable to the situation where order delivery is executed using k vehicles with a specified discount $d_i^{(k)}$ relative to the basic delivery cost kC_{0i}.

The decision on whether to use formula (7) or (13) in calculations must be clarified when determining the order size within the optimization procedures. Naturally, the formula for the interest rate should correspond to the actual loading situation of each vehicle of i-type. Specifically:

1. If each vehicle of i-type is loaded up to its maximum allowable capacity (i.e., $q_i^{*(k)}/k = q_{mi}$).
2. Or if each vehicle of i-type is partially loaded (where the load per vehicle is, on average, determined by $q_i^{*(k)}/k = q_i^{(k)}/k = \sqrt{2\left(1 - d_i^{(k)}\right) DC_{0i}/(kC_h)}$.

The actual situation can be readily identified based on the analysis of the optimization results applicable to the specific supply chain.

4. Constraints Imposed by Delays in Revenue Receipt

When the value of P_P is small and the values of C_{0i}, C_h, and L_P are large, it may not be feasible to cover the previously noted payments solely from revenue. In other words, during modelling and optimization, we must consider that the accumulated revenue during the reorder interval might be insufficient to pay these specified amounts directly from revenue.

In such cases, equations (7) and (13) will not accurately reflect the return on working capital. Therefore, it is essential to identify the class of models for which the recommendations developed here for estimating the profitability indicator of working capital utilization are applicable.

Conditions that need to be imposed on the model parameters to ensure that working capital can indeed be considered as the monetary amount equal precisely to the costs of delivering the goods—while assuming other payments are made from revenue—have been analysed in previous studies [4][5][21]. One approach assumes it is sufficient to ensure that the allowable delay in receiving revenue does not exceed the average time interval until revenue is received from the delivered order.

Here, we propose an alternative approach to simplify the analysis of the allowable delay in revenue receipt Ω. Since we are considering a deterministic model with constant demand, exactly $q_i^{*(k)}$ units of goods will be sold during the reorder interval. Thus, the revenue $V(t)$ as a function of time increases linearly, starting from time Ω (due to the delay in receiving revenue from the sold goods), rather than from the moment the goods are delivered.

Over the initial time interval $t\epsilon[0,\Omega]$, the revenue function equals to zero. For the subsequent interval $t\epsilon[\Omega, 1+\Omega]$, the revenue function is defined as $V(t) = (t-\Omega)\cdot[D(C_P+P_P)]$.

To ensure business efficiency, the revenue from each order delivery must exceed the costs associated with operating the modelled supply chain. This includes costs directly related to the delivery of the order, such as holding costs and deductions from revenue made with each delivery. Therefore, the following inequality (14) must hold for the model under consideration:

$$q_i^{*(k)}\left(P_P - L_P - \frac{C_h q_i^{*(k)}}{2D}\right) > \left(1-d_i^{(k)}\right)kC_{0i}. \quad (14)$$

This inequality ensures that the revenue from the delivery will cover the payments related to the reorder interval.

An important aspect of the modelling procedure is specifying the moment when it becomes possible to pay from revenue the cost of the first order and it`s delivery $V(t) = q_i^{*(k)}C_P + \left(1-d_i^{(k)}\right)kC_{0i}$. To make this payment, the certain time (t_{spec}) is required:

$$t_{spec} = \Omega + \frac{\left[q_i^{*(k)}C_P + \left(1-d_i^{(k)}\right)kC_{0i}\right]}{[D(C_P+P_P)]}. \quad (15)$$

To use the previously provided equations (7) and (13), the deferral in payment of the order cost must be agreed upon in advance.

All other payments mentioned above, which need to be paid after the delivery of the order, can also be covered by the received revenue. A deferral may be required for these payments as well. Therefore, it can be assumed that the investments required for operating the supply chain during each reorder interval actually include only the costs of delivering the order, amounting to $\left(1-d_i^{(k)}\right)kC_{0i}$.

It is straightforward to formalize the conditions that guarantee the payment of the order cost can be made from revenue. If inequality $t_{spec} < T_i^{(k)}$is met, even if there are delays in incoming payments, the revenue from the next delivery will be enough to cover the costs of placement the next order (including delivery costs). Otherwise, the revenue from product sales may not have been received by the time payment is due. And the corresponding deferral in payment of the order cost must be agreed upon with supplier. The duration of the agreed-upon payment deferral must be at least $t_{spec} - T_i^{(k)}$.

Additionally, the possibility of deferring the payment of holding costs must be arranged. Furthermore, for the model considered here, inequality (14), corresponding to the efficiency of the modelled supply chain, must be satisfied.

5. Numerical Illustration of Simulation Results

To illustrate the proposed optimization procedures, we will apply them to estimate the optimal order size for a product. The following initial data is provided:

- $D = 504$ (annual demand in units);
- $C_P = 400$ (USD, cost of product per unit);
- $P_P = 260$ (USD, profit from sale per unit);
- $L_P = 150$ (USD, deductions from profits per unit);
- $C_h = 10$ (USD, storage costs of unit per year);
- $\Omega = 0.01377$ (years, allowable delay in receiving revenue from sold products, equals to 5 business days);
- $C_{01} = 2,100$ (USD, cost of one delivery using vehicle type 1);
- $C_{02} = 1,650$ (USD, cost of one delivery using vehicle type 2);
- $q_{m1} = 21$ (capacity limit of vehicle type 1);
- $q_{m2} = 16$ (capacity limit of vehicle type 2);
- $d_1^{(2)} = 14\%$ (the discount on the cost of one delivery using 2 vehicles type 1).

As we can see, we have two types of vehicles (1 and 2) with different delivery costs and capacity limits. Furthermore, when using two vehicles of type 1, the carrier offers a discount on the transportation cost.

Note that in this way, one can compare both the vehicles (as in current example) and the carriers themselves by setting C_{0i} as the cost of the forwarding services relative to different cargo volume options q_{mi}.

To proceed with the optimization procedures, we need to determine the values of $q_i^{(k)}$ for all available vehicle types and their various quantity configurations. As noted in [9] – [10], using multiple vehicles is advisable only when discounts are available. Therefore, we will consider three different delivery options:

$$q_1^{(1)} = \sqrt{2 \cdot 504 \cdot (1-0) \cdot 2100/10} = 460.09;$$
$$q_1^{(2)} = \sqrt{2 \cdot 504 \cdot (1-0.14) \cdot 2100/10} = 603.40;$$
$$q_2^{(1)} = \sqrt{2 \cdot 504 \cdot (1-0) \cdot 1650/10} = 407.82.$$

Let us calculate the values of $q_i^{*(k)}$ values using formula (3) for each option. This is necessary for two purposes. Firstly, to verify whether condition (14) regarding the effective utilization of working capital is satisfied, and secondly, to determine which of the two scenarios (Section 3.1 or Section 3.2) each option corresponds to:

$$q_1^{*(1)} = 21;\ q_1^{*(2)} = 42;\ q_2^{*(1)} = 16.$$

Now we need to verify whether the order payments can be covered by revenues. To do this, we use inequality (14) for each of the options $q_1^{*(1)}, q_1^{*(2)}$ and $q_2^{*(1)}$. It is important to use the corresponding values of $d_i^{(k)}$, k, and C_{0i}.

For $q_1^{*(1)}$ is will be: $2,306 > 1,806$. Condition is satisfied. Similarly, the values for the remaining options also satisfy the inequality (14). So, we should further analyse all of the options.

Note that if the condition is not satisfied for any of the options, we can exclude those options from consideration, since it would not be possible to make payments from revenue.

Now we will finally determine the values of $r_i^{(k)}$ for the options under consideration. Since $q_1^{(1)}$, $q_1^{(2)}$ and $q_2^{(1)}$ meet the condition $q_i^{*(k)} = kq_{mi}$ (the order size fully utilizes the vehicle's cargo capacity), we will use formula (7). Thus, we obtain the following estimates of profitability:

$$r_1^{(2)} = \frac{504 \cdot (260-150) - \frac{504 \cdot 2100 \cdot (1-0.14)}{21} - 2 \cdot 21 \cdot 10/2}{2 \cdot 21 \cdot 400 + (1-0.14) \cdot 2 \cdot 2100} = 58.23\% ;$$
$$r_1^{(1)} = 47\%;\ r_2^{(1)} = 42.05\%.$$

Therefore, due to the available discounts, the best option is $q_1^{(2)}$ (using two vehicles of type 1), which has the highest profitability value of 58.23%.

At the same time, the interval between deliveries will be $T_1^{(2)} = 42/504 = 0.08\ (years)$. Namely, about 30 days (one delivery per month).

This numerical simulation illustrates that the goal of the study has been achieved: we can use profitability estimates to determine the optimal inventory management parameters for a specific product. Moreover, by using Equation (15), we can now determine the duration of the payment deferral that we need to negotiate with the supplier:

$$t_{spec} = 0.1377 + \frac{42 \cdot 400 + (1-0.14) \cdot 2 \cdot 2100}{504 \cdot (400+260)} = 0.075063.$$

The obtained value of $t_{spec} = 0.075063$ indicates that we will receive revenue equal to the investment in the order and its delivery after 28 days.

In our case, we would have time to recover the costs invested in the order and its delivery within one order cycle (because $t_{spec} < T_1^{(2)}$).

However, if $t_{spec} > T_i^{(k)}$, it would be necessary to further coordinate the duration of the delay in contract with our suppliers for a period equal to the difference between t_{spec} and $T_i^{(k)}$.

6. Conclusion

This study emphasizes the critical role of accurately estimating the interest rates when optimizing inventory management systems that incorporate the concept of the TVM. These interest rates should characterize the profitability of the modelled supply chain. Naturally, this requirement applies to the modifications discussed in this paper, which account for allowable delays in revenue receipt from delivered orders.

This research presents an analysis focused on estimating the required interest rates. These estimates provide managers with the ability to account for several practical factors, including:

- The type and number of available vehicles;
- The capacity of vehicles;
- The discounts for multi-vehicles deliveries;
- Delays in payments from sold products and the impact of these delays on profitability and the ability to pay for orders using revenue;
- The specifics of working capital, along with corresponding changes in the profitability metrics of the modelled supply chain.

The analysis has been applied to a specialized EOQ formula format, yielding the following key results:

1. Formulas for determining the profitability rate, which depend on vehicle capacity (fully utilized or only partially), considering relevant factors;
2. A mathematical framework for determining the payment deferral period to be specified with a supplier, factoring in delays in incoming payments.

Thus, the results of research enable the determination of optimal decisions regarding order size, the type and number of vehicles required for transportation, and the payment deferral period to be negotiated in a contract.

The provided numerical illustration demonstrates the effectiveness of this approach, its practical applicability and potential benefits in real-world supply chain scenarios. Given the current trends in payment delays, the use of the results obtained may help to make optimal decisions regarding inventory management.

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